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A RESPONSE EVALUATION APPROACH: AN AID FOR COMPUTER ASSISTED INSTRUCTION LESSON WRITING

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THESIS

A Response Evaluation Approach:
An Aid for Computer Assisted
Instruction Lesson Writing

by

Fred D. Soetekouw

September 1980

Thesis Advisor: R. H. Weissinger-Baylon

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A Response Evaluation Approach
An Aid for Computer Assisted Instruction Lesson Writing

by

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ABSTRACT

This study develops a computer assisted instruction response evaluation technique to aid CAI lesson writers in course development. The objectives for this technique was to experimentally identify principal incorrect response forms and determine characteristic error patterns for a given concept to be taught. Using a specific concept, the proposed technique was evaluated. Sixty-one subjects were randomly selected from three different universities and assigned to solve a problem-solving task. Using mental imagery and oral reporting, they produced the necessary protocol for this experiment. Although the response evaluation technique developed in this study was reasonably successful, a specific response error pattern for each subject could not be determined.

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ACKNOWLEDGEMENT

To my parents and my wife Pu Hui

I. INTRODUCTION

Abrams (1971) asserts that the most difficult problems impeding the progress of computer-assisted instruction (CAI) are those involving the basics of the educational process itself and the inherent complexity of CAI from the point of view of the educators. While a good CAI system will not appear complex to the students and the instructors who use it, the existing and proposed new system are difficult for even knowledgeable educators to evaluate for applicability and desirability.

Even more basic is the problem of the use of diverse forms of responses to indicate the attainment of knowledge and understanding. This problem is associated with two questions:

(1) What type of response is required from the student and the CAI system for a given educational evolution?

(2) Will this response be a true indicator of the student's understanding of the concepts involved?

These are difficult, nevertheless, important questions as indicated by Simon: "the possibilities that are opened up for improving human problem-solving, thinking, and decision-making activities" are "at least as important" as the genuine revolutionary "developments [which are now apparent] in automation." [Weissinger-Baylon, 1978, p. 1].

The author shares both Abram's concern and Simon's confidence for this intrinsic problem area of computer-assisted instruction. Although much has been accomplished in this problematical area, much remains to be solved. Until the many factors involved in the response evaluation process is more fully explored and more thoroughly understood, it is the author's opinion that it will be difficult for CAI to realize its full potential.

There have been many variations in the definition of computer-assisted instruction (CAI) in the past two decades. The one definition that encompasses the whole field and which will be used in this paper is best defined by Abrams:

> . . . the use of computers to aid teachers and students in the educational process, utilizing such functions as presenting materials and problems, guiding a student's path through a course by selecting the material to be presented or by assigning tasks to be completed away from the computer, or any combination of these functions.
>
> [Abrams, 1971, p. 4]

Therefore, CAI does not just simply imply typing out questions to which students respond and by then giving cues and different hits, CAI includes the computational use of the computer, when appropriate, as part of the instruction.

Computer-assisted instruction systems may be generally viewed in two ways: machine-directed and student-directed. In the machine-directed approach, various paths and alternatives through instructional material is programmed into a machine. In this approach there must be prior knowledge of

the needs of the student and a predetermined method by which the computer can evaluate the student's current state of knowledge. In addition, the student's past performance record and techniques for describing a further path for each student, which would optimize his learning experience, must be at the machine's disposal.

In the student-directed approach, the sequence of the material presented is altered only at the request of the student. The approach uses the aspect of learning by discovery and experimentation. This area, however, has been almost untouched by current research and, therefore, much less has been done to utilize this type of approach.

This paper deals primarily with the machine-directed CAI approach. A new computer-assisted instruction (CAI) technique with the potential to improve the effectiveness of CAI was developed and evaluated. This technique experimentally identifies the principal incorrect forms of any given concept that is taught. Characteristic error patterns are associated with incorrect forms so that the subject may be presented with a CAI response tailored to that particular incorrect concept that he has acquired. New instructional material or other techniques could then be utilized depending upon the student's understanding or lack of understanding of specific concepts.

Absence of any contributing literature in the area of CAI response evaluation and personal communications with

prominent individuals in the area of CAI, indicate that the approach discussed in this study, is the first significant attempt to provide a specific set of rules or guidelines for evaluating the subject's responses. This does not imply that the response evaluation area of CAI was never considered or discussed, but that a set of rules has never been formalized. The following method has therefore been designed to provide CAI lesson writers with an additional tool for structuring CAI course material.

II. BACKGROUND

A. THE NEED FOR A RESPONSE EVALUATION TECHNIQUE

The use of various computer-assisted instruction systems has been on the upswing in the last decade. This growth has been desirable and is a result of the increased amount of human knowledge that needs to be attained, the ever increasing number of students to be taught, and the availability of qualified instructors. Early studies revealed that consistent time savings occurred during a three year experiment using microcomputers to manage students' self-paced progression through a course (Evans and Johns, 1979). Orlansky and String (1979), in an iconoclastic attack on the cost-effectiveness of thirty studies of military computer-based education (CBE) have also concluded that a thirty percent time savings is possible. It can therefore be generally agreed that CAI can achieve educational objectives more efficiently and more effectively than other teaching methods. However, four basic educational factors accentuate the need for a computer-assisted instruction (CAI) response evaluation technique.

The first factor is the trend toward individualized instruction. The fewer the number of students for which a teacher is responsible, the more time and individual attention each student will receive. Therefore, from a

theoretical aspect, the more each student will gain from his educational experience. Conventional mass media such as books, films, television, and even programmed instruction are inherently incapable of individualization. Books, films, and television programs would have to be rewritten to fit each student, and conventional programmed instruction systems lack the flexibility and decision-making capability required of a truly effective individualized system. CAI seems to have the potential to meet these requirements. However, the current relatively sophisticated computer-assisted instruction systems are only tailored towards students who are homogeneous in background, ability, cognitive style, and motivation. The systems are incapable of individualization in the sense of structuring a response as a true indicator of the student's understanding of the concepts involved. However, by evaluating the subject response and then providing a tailor-made CAI response to reflect understanding or lack of understanding of a given concept, seems to have the potential to improve the effectiveness of CAI and its value to individualized instruction.

The second factor is the evaluation of the student's learning progress. Although, sophisticated CAI systems keep track of the number of wrong responses for a particular concept, they do not identify to the educator the incorrect concept that the student learns. Using a response evaluation technique, the wrong concept may be identified

and new instructional material, review material, drill and practice sessions, tutoring, or other techniques could then be utilized depending on the student's understanding.

The trend towards the use of microcomputers is the third factor. Computer-assisted instruction has typically been applied using large, dedicated computer systems and special CAI languages. There are virtually no limitation on memory or file sizes for the instructional material. However, microcomputers are now being sold at prices comparable to color television sets and their use in educational settings will skyrocket in the next few years. Invariably, CAI will be one of the uses to which these computer systems are put. These systems, however, have inherent limitations. The most important being memory size. Since, these limitations are fixed attributes of the systems, they must be dealt with creatively. One method is to use a response evaluation technique. Using such a technique, the size of the programs may be reduced by limiting the required number of paths or alternatives through the instructional material.

Finally, the fourth factor is the limited applicability of computer-assisted instruction to university level subjects. Weissinger-Baylon asserts that according to Atkinson, "the relatively sophisticated computer-assisted instruction systems that have been successfully introduced for reading and elementary areas are neither applicable to under-graduate mathematics nor to other university level subjects; too little is known of the structure of the required skills

or of the psychological processes for their mastery." Until the type of response for certain concepts are more fully explored and more thoroughly understood it will be difficult for CAI to realize its full potential.

B. RESEARCH OBJECTIVES

The most difficult problem impeding the progress of computer-assisted instruction are those involving the basics of the educational process itself. The type of response that must be obtained from the student to get an indicator of his true understanding of the concept, is a difficult question. Although there has been some work performed by Newell, Simon, Shaw, and Weissinger-Baylon in the area of human problem solving [Weissinger-Baylon, 1978, p. 5], it has only been applied to the area of computer-assisted instruction in a very limited amount. Substantial progress in this direction, however, will permit far reaching new approaches to education.

The present study is conceived as one step in a longer program, the development of major improvements, efficiency and value of computer-assisted instruction systems. The immediate objective is to develop and evaluate a new computer-assisted instruction (CAI) technique using mental imagery and verbal responses to experimentally identify the principal incorrect forms of a given concept that the students usually learns. Characteristic error patterns will be identified and associated with incorrect concept forms on a diagnostic test and will be evaluated in regards to its application to a computer-assisted instruction system.

C. TASK SELECTION

Task selection for the development and evaluation of any new technique or procedure must be based on many factors and therefore makes it a difficult chore. Weissinger-Baylon, from his research, states that the most important factors in task selection are: "emphasis on tractability or support for the experimenters hypothesis must be minimized; the research must have application, at least potentially; and finally, the complexity of the task must not limit the communicability of the findings." The author agrees with Weissinger-Baylon's findings, but adds that a task may be selected because of it's interest to the analyst and because it complements the work of others.

The choice for the selection of mathematics as a field to test and evaluate responses for a CAI system was made as a result of its many advantages: "for the chosen problem area, both concept and vocabulary are highly developed; adequate intellectual resources are available for this topic; its method and administrative procedures are convenient; and it is an important area of application, controlling almost all entries to attractive professions."

Determining a problem area within the chosen field was the next stage in the selection process. It was a difficult task. The material had to satisfy both the requirements of meeting the objectives of this research and, most importantly, be within the grasp of existing information

processing methodologies.

Weissinger-Baylon's personal communication with Karl deLeeuw [Weissinger-Baylon, 1978, p. 30] , proposed an analysis of a group of closely related concepts having to do with "bounding" of sets. It was felt such a concept would adequately propose or illustrate, at least theoretically, the technique to evaluate and analyze responses for a CAI system. The problem that arose, however, was what type of bounding concept to use--the greatest lower bound, the least upper bound, or determine if the set is bounded above or below.

Preliminary studies conducted by Weissinger-Baylon on the proposed concepts, suggested that the focus of the analysis for a response evaluation process for a CAI system, be narrowed to a single definition. His studies on the upper bound concept produced the most interesting results and it was determined that the importance of the upper bound concept, its manageability, and Weissinger-Baylon's prior research in this area, made this concept the most attractive candidate for this research study in evaluating the response process for a CAI system.

D. METHODOLOGICAL OVERVIEW

The development of a suitable methodology proceeded in stages. Test results from a research study conducted by

Weissinger-Baylon on the upper-bound of sets concept, were made available for this research, and pre-empted the need for duplicate and redundant testing in the initial phase of the study.

1. Subjects

The sixty-one students who served as subjects were volunteers from three different universities: the Naval Postgraduate School, Stanford University and Colorado State University. Subjects were selected to provide a reasonable balanced representation by sex, specialization and professional levels. They were obtained from bulletin board announcements, distribution of sign-up sheets and by personal request. With the exception of the Stanford University students, who received a $6.00 fee, no other compensation was offered for the subject's participation in this experiment. The mix consisted of 23% females and 77% males. Of these, 33% were graduate or undergraduate students in the mathematical sciences (Computer science, Statistics, Operations research and Mathematical sciences) and 67% were undergraduates or non-mathematical science graduates. Prior research had dictated that subjects without university mathematics would experience difficulty with this particular concept and were therefore required to have had at least two years of calculus.

2. Design

The problem-solving task was initiated by looking at fifteen different, but related definitions from advanced calculus. From the results, a final version was produced, which focused on one definition: the upper bound of a set of real numbers. The criteria for an upper bound of a set was met if there existed a real number which was greater than every element of the set.

Nine problems were chosen from standard texts of advanced calculus and grouped into the following four types of sets:

(1) finite sequences: e.g. {1,2,3,4}

(2) intervals: e.g. (a,b)

(3) convergent infinite sequences:
e.g. {1, 1/2, 1/3, ...1/n,...}

(4) divergent infinite sequences:
e.g. {1,2,3,...n,...}

[Weissinger-Baylon, 1978, p. 32]

In addition, the above sets were presented in the following methods:

(1) enumerative: {1,2,3,4}

(2) nominal: N, the natural numbers

(3) attributive: finite set

[Weissinger-Baylon, 1978, p. 32]

The sets were randomized by type and presentation method.

These representations vary slightly from the final design since they were modified to make them suitable for a

wide range of subjects. Further, it is well to note that the degree of accuracy will be based on the degree to which the subject's responses match the existing textbooks. The production of imagery or step by step analogy of the problem set was not consciously a criterion.

3. Procedure

The detailed, step-by-step procedure for the CAI response evaluation technique is given in chapter 3. The procedural model, as designed, consists of seven major components: selection of target content, design of problem-solving task, selection of protocol methodology, pretesting, refining the task, administering the final version of the task, and analysis of results.

The procedural methodology that was used for administering the problem-solving task consisted of contacting the subject prior to the experiment and agreeing on a set time for administration of the task. Before beginning the problem-solving task, the subjects were interviewed to determine their qualifications for testing on the particular concept. In addition, during the interview the subjects provided the following information: year,

department, and educational and career plans. When the participant was found to be qualified, he was given the following instructions:

> Instructions. On the next page, you will find a definition taken from an undergraduate mathematics course. At first the language may seem imposing, but the concept is not difficult. Please read carefully, and try to understand it as well as you can. Next, go on to the practice problem and the questions which follow:
>
> We would like you to think out loud as much as possible. If you use any mental imagery in your thinking, please draw or describe it. At the end of each problem, you are to write a complete summary of your thought process.
>
> You may begin when you wish. There is no time limit. Please ask any questions which you feel are necessary. Do not forget to think out loud, or draw or describe any mental imagery you may have.
>
> [Weissinger-Baylon, 1978, p. 36]

A tape recorder was then put in front of the subject to report his oral responses, if any occurred. When the subject indicated that he understood the instructions, he was given the test booklet and then asked to start the exam. The examiner was seated beside the subject to observe and take notes. At the end of the entire session, the subject was again interviewed to clarify and explain the exam and, in some cases, uncover errors in the written protocol without the risk of biasing the responses.

In summary, the administration process was designed to systematically obtain data using the following protocol methodology:

1. Verbal protocol--to clarify infrequent ambiguities in the written summaries.

2. Graphic protocol--to illustrate a path of the subject's thought processes.

3. Written summaries--to obtain a formalized, step-by step analogy of the problem.

4. Interaction with experimenter--to obtain additional clarification when imagery explanations were ambiguous and when steps were omitted during the written summaries.

The importance of this data collection method is demonstrated in the data analysis phase of the procedural model.

III. DEVELOPMENT OF A RESPONSE EVALUATION TECHNIQUE

A response is a unit of behavior and the building block of complex performances. The flick of an eye and the twitch of a finger are examples of simple responses; eating, walking, speaking and reading are all instances of more complex responses. A primary objective of educational technology is the guidance of individual's responses. To accomplish this objective, the educator must first define and enumerate the components of the performance, that is, the responses, that he wishes to produce. It is then possible to arrange the stimulus conditions which will result in the desired response. It also becomes possible to develop objective measures of the frequency and accuracy of the response.

The consequences of a student's response are extremely important in learning. The events which follow the occurrence of a response have an effect upon future behavior. Examples of such response consequences are reward, punishment, and knowledge of results. The reward or other stimulation provided to the learner immediately after responding is a significant factor which seems to determine whether learning takes place. The occurrence of certain consequences of behavior that are effective in producing and maintaining behavior is called reinforcement by behavioral psychologists. In an educational situation, the instructor

controls the consequences of the student's behavior. Since these consequences determine whether the student learns, the educator will want to maximize those consequences which facilitate learning.

Definition of an effective and efficient procedural model for a response evaluation methodology began with an analysis of the characteristics of the instructional environment. This examined specifically the environment defined by the evaluation of course mastery, and the types of instructional problems for which a response evaluation process could provide solutions in this environment.

The procedural model, as derived, consists of seven major components, each with a number of constituent steps. The components are selection of target content, design of problem solving task, selecting protocol methodology, pretesting, refining the task, administering the final version of the task, and analysis of results. The full model is shown in figure 1. The model's seven component steps are described in the following paragraphs.

A. SELECTION OF TARGET CONTENT

The first component of the procedural model (see figure 2) assumes the presence of a main track of non-CAI materials and defines criteria for selecting those materials for which alternative CAI modules are to be developed. The steps in this component do not constitute a precise algorithm for

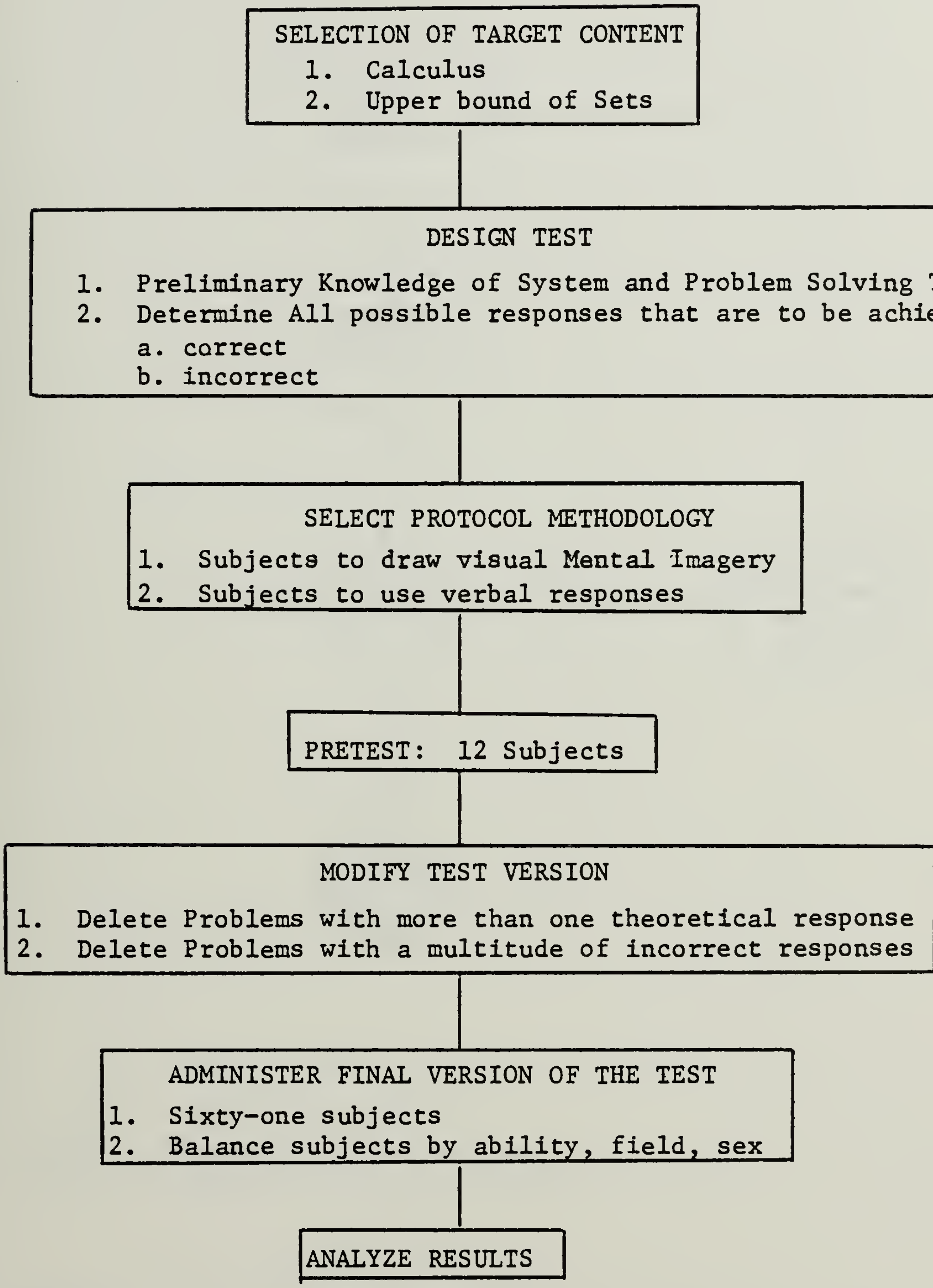


FIGURE 1. This flowchart summarizes the main steps in the experimental design and administration of a response evaluation procedure.

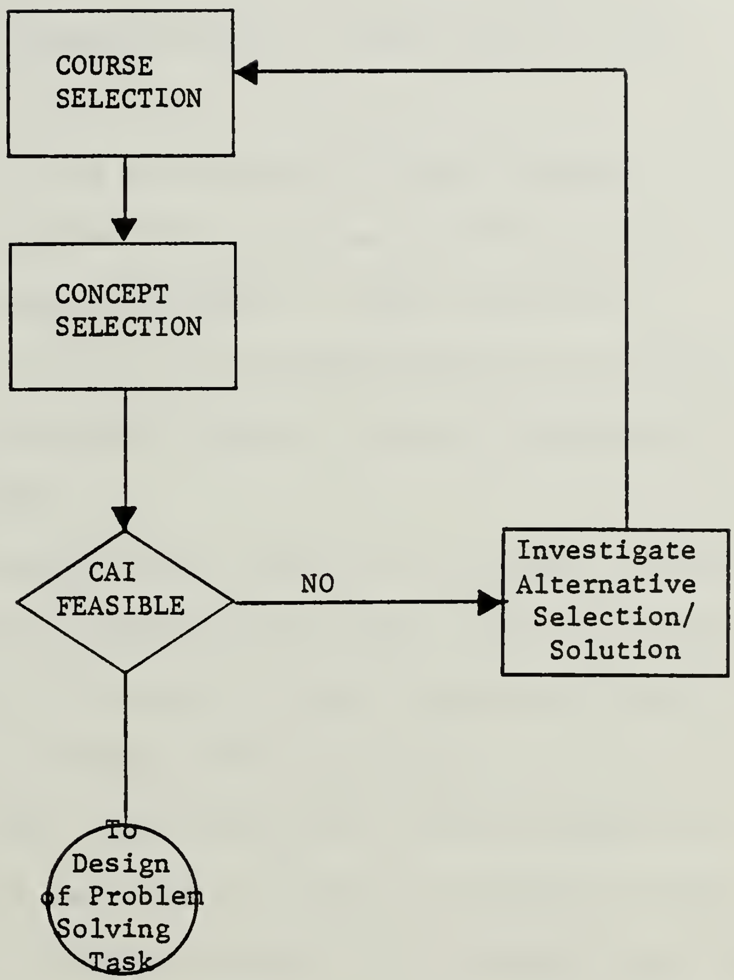


FIGURE 2. Selection of Target Content

content selection. Rather the intent is to identify a number of factors which should be considered in determining target content selection. Few hard and fast rules can be defined, and value judgements play a major role in the decision process.

The first step in the selection of target content is to select a course or courses where the use of CAI will be beneficial to the training process. This should be a school-level decision based on considerations as training cost per student, instructor student ratios, and types of training materials used.

The principal consideration must be to choose a course where CAI materials can improve motivation, and where the unique capabilities of CAI will provide needed information.

After selecting a course, the next step is to select a specific concept. Here, the selection considerations are the importance of the particular concept, concept failure rates, variability of concept mastery test scores, and average times and variability of times to complete the concept.

When the two selection steps (course and concept) are completed, it must be determined if CAI is feasible for the tentative selections. This is where preliminary knowledge of the system plays an important role. Possible reasons for determining non-feasibility of CAI include necessity for graphics or simulations which are beyond the capabilities of the available system and personnel, or too few concept

selections to justify hardware or development cost.

With the final determination of CAI feasibility, the educator is now able to define and enumerate the components of performance, in other words, the response he wishes to produce for a given task. The educator is ready to begin the next component of the procedural model, the design of the problem-solving task.

B. DESIGN OF PROBLEM SOLVING TASK

The steps of the design and the development of a problem-solving task component is shown in figure 3. It assumes that the course and concept selection together with their CAI feasibility have already been determined.

In starting to develop a new teaching session, an educator must be given a certain amount of basic preliminary information about the material and system to be used. This may be provided in the form of text, drawings, diagrams, moving pictures, or any combination of these modes. This process will ensure that the finished materials avoid such problems as omitting necessary content, covering objectives incompletely or at an incorrect level, incorrect or inadequate emphasis on particular subjects or concepts, incomplete material supplements, poorly designed teaching strategies, and tests which do not address the objectives. Once the workings of the system and the basics of the materials are understood, development of the task to obtain

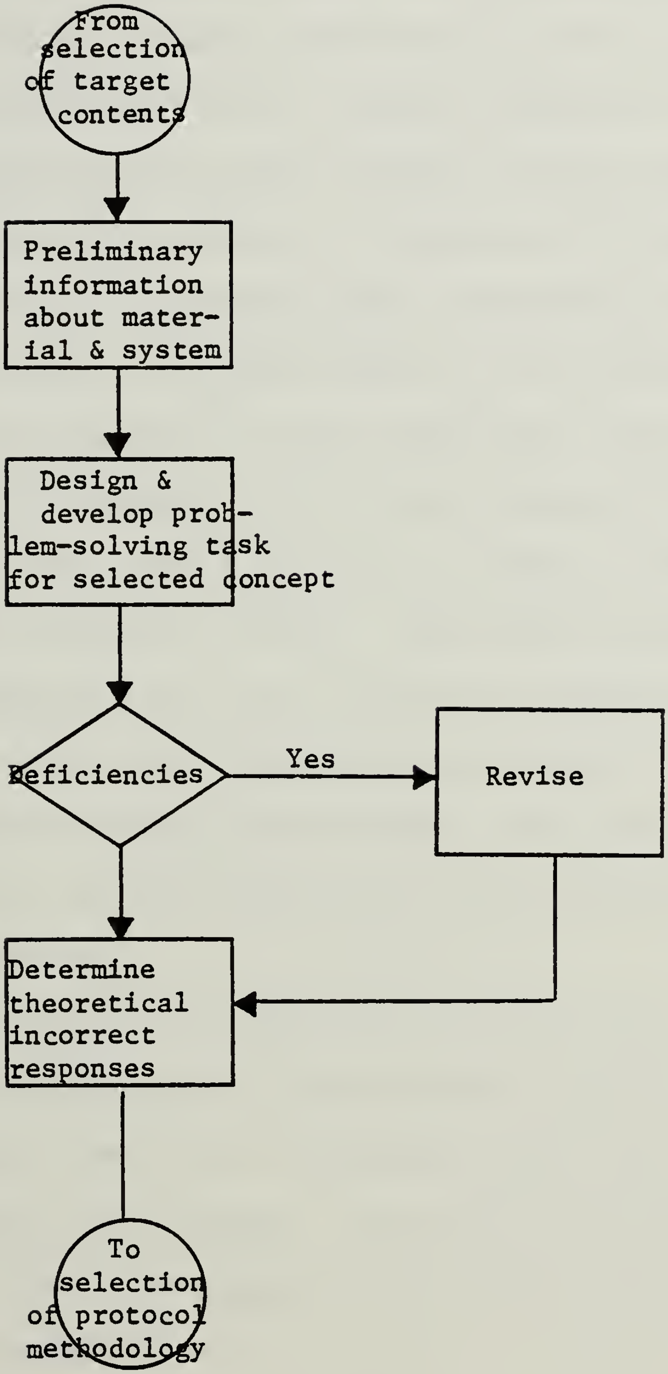


FIGURE 3. Design of Problem Solving Task

proper responses from the user and the system may begin.

After this initial phase, the design of the problem-solving task should begin with a careful analysis of the instructional requirements. The forming of the instrument to evaluate a particular concept is obtained through the educational process of an instructors preliminary knowledge of the subject matter. In this phase, it is important that the test has face validity for the objectives which addresses the particular concept. It is also important that the problem-solving task cover the specific concept matter at the appropriate difficulty levels. Since this instrument is the principal element which will guide the response evaluation process, it must fully represent the intent of the educator to assure adequate performance. Any revisions must be reviewed by the educator to insure the technical accuracy of the content changes.

Unless these first two steps are successfully completed, all subsequent efforts may be jeopardized. For example, if the problem-solving task is so designed that it will not evaluate the particular concept, the work in the evaluation of principal incorrect responses for that particular concept will be largely wasted.

The final step of the design of the problem-solving task includes the determination of the proper responses desired and the possible major incorrect responses that may be

obtained. Since, the number of possible incorrect responses for any particular concept is theoretically infinite, this process will only serve as an initial starting point in the identification of the principal incorrect forms of that concept that is taught.

C. SELECTION OF PROTOCOL METHODOLOGY

The protocol methodology for the administering and evaluation of the problem-solving task (see figure 4) are given as a set of rules. Revision of the protocol may be made at any time it is determined that it is non-feasible for a particular application. The rules are as follows:

1. Selection of students for use on the problem-solving task should be on a volunteer basis. Using volunteers rather than individuals ordered to participate in the problem-solving task will avoid some possible attitude problems.

2. Volunteers should be selected on their qualification for the particular concept being tested. They should be selected to provide a reasonably balanced representation by sex, specialization and professional level.

3. The problem-solving task should be conducted outside the classroom and should be informal. Attempting to work within the classroom is likely to disrupt regular classroom activities. In addition, it has been found that the problem-

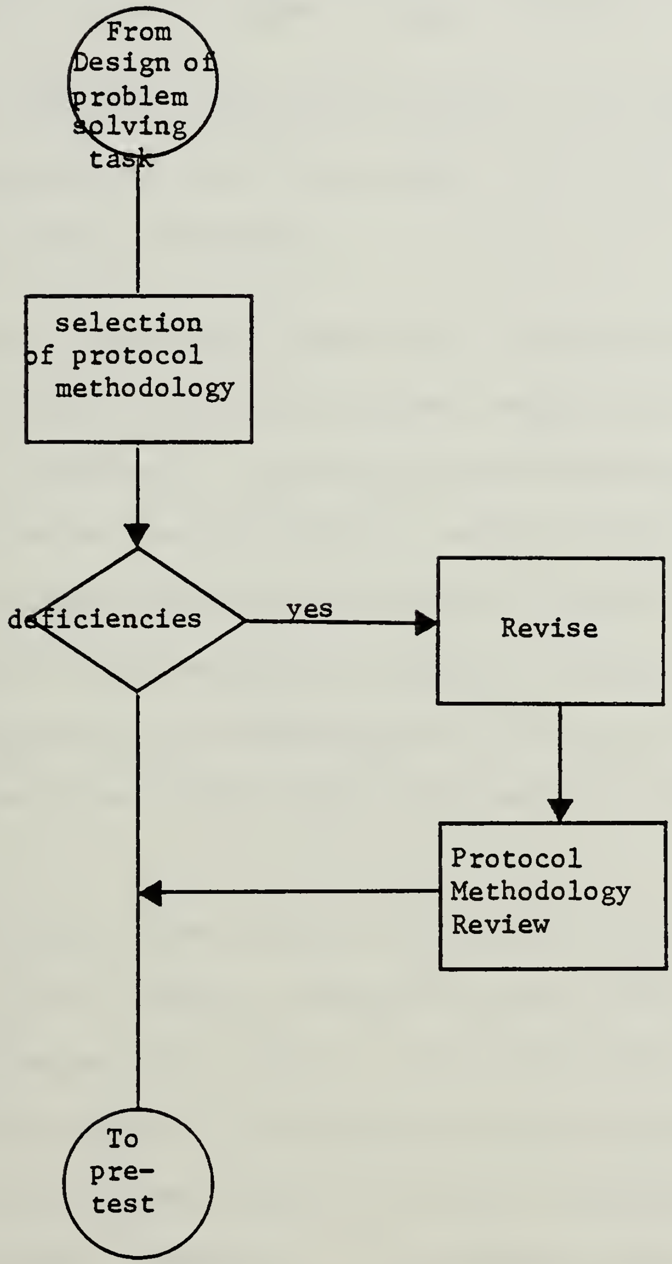


FIGURE 4. Selection of Protocol Methodology

solving task should be administered to one test subject at a time. Group testing tended to inhibit the subject's willingness to verbally report their thought processes.

4. The examiner should be available to answer questions, provide guidance where necessary, and note areas where students have difficulty.

5. Finally, and most important, the test subjects are to learn the particular concept and apply it to a number of practice problems. During the problem-solving task, the test subjects are to verbally report their thought processes and draw and describe mental images, if any occurred. In this way, the subject's thought processes for a particular concept may be determined by analyzing the result and the sequence of steps that were used to obtain that result.

The above rules provide effective guidelines for an efficient and standard administration and evaluation of the problem-solving task. It is noted that verbal reports of a subject's thought processes may be tape recorded for accuracy and historical references. This is only required, however, if it has been found that test subjects tend to lean towards verbal reporting vice the written method. Continual evaluation of the methodology must be made during the problem-solving task for any particular situation.

D. PRETEST

The fourth component of the procedural model (see figure 5) assumes that a dedicated amount of time has already been spent in the selection and development of a problem-solving test. The purpose of this phase is not so much directed at evaluating the response effectiveness of the problem-solving task as it is at locating errors of content and logic which were overlooked. Typically, the number of students tested would not be sufficient to answer questions concerning the incorrect concepts that the student learns, but do provide an avenue by which the educator can learn if the materials are easily understood by their intended audience. The pretest provide answers to questions such as the following. Are there inconsistencies between the concept and the problem-solving task? Are directions to the student clear enough to be followed? Is wording clear? Is the problem-solving task too long to hold the student's attention?

The pretest should generally be administered to 12-15 students, but the number will vary with the number of errors found early in the pretest. If many errors are found with, for example, the first four or five students, it would be preferrable to suspend the pretest until the problems have been resolved. If available manpower permits, it may be desirable to supplement review of the pretest by other instructional personnel for the purpose of locating grammatical and

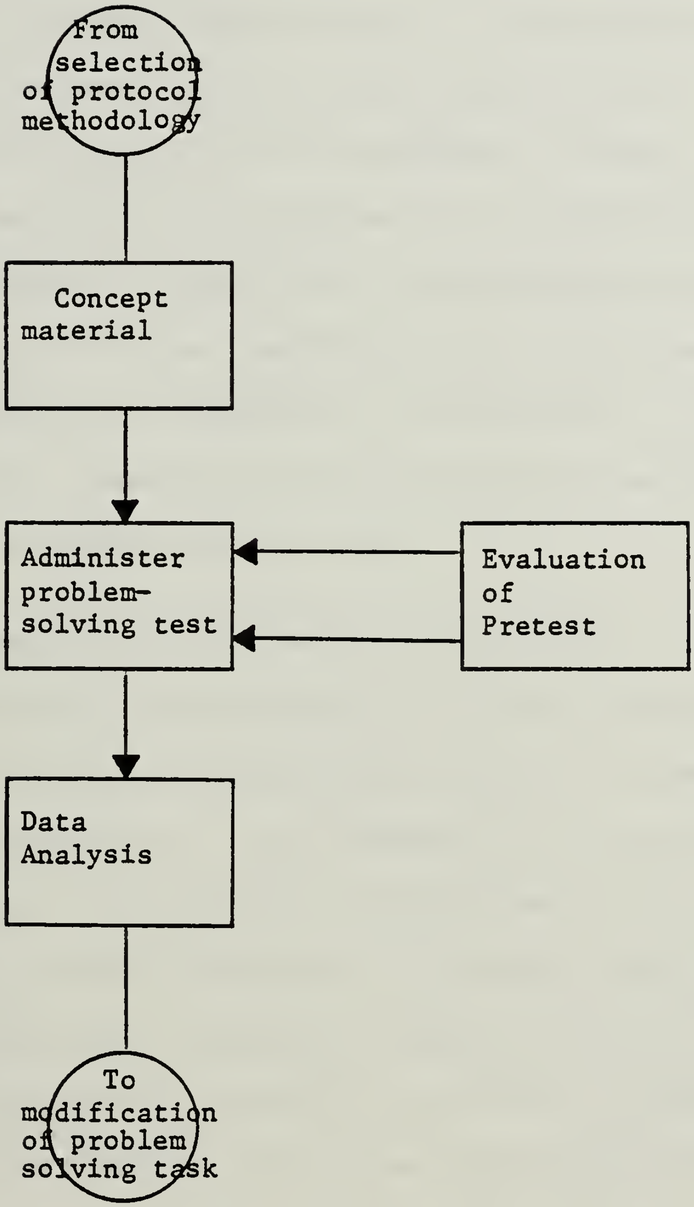


FIGURE 5. Pretest

typographical errors and problems with clarity.

The next step pertains to data analysis. With the use of imagery, as specified in step 3 of the procedural model, principal incorrect forms of the concept that a student learns may now be initially determined. These incorrect concept forms are to be compared to the theoretical incorrect forms established in step 2 of the procedural model. After comparison of both the theoretical and diagnosed forms of the principal incorrect responses, the educator is now ready to begin the next component of the procedural model, modification of the problem solving task.

E. MODIFICATION OF THE PROBLEM SOLVING TASK

This component (see figure 6) of the procedural model is designed to modify or correct any errors and ambiguities revealed by the pretest. It's purpose is to evaluate the instructional effectiveness of the materials to determine the specific areas of weakness. Problems which have more than one theoretical incorrect response or too many diagnosed responses, such that a specific incorrect concept cannot be applied to the problem, are to be evaluated and deleted from the problem-solving task. If it is found that the problem-solving task has too many unresolvable problems, it must be determined if the particular subject matter is actually testable. If not, the procedural process for a response evaluation procedure terminates at this point, otherwise the educator is ready to begin the next component of the

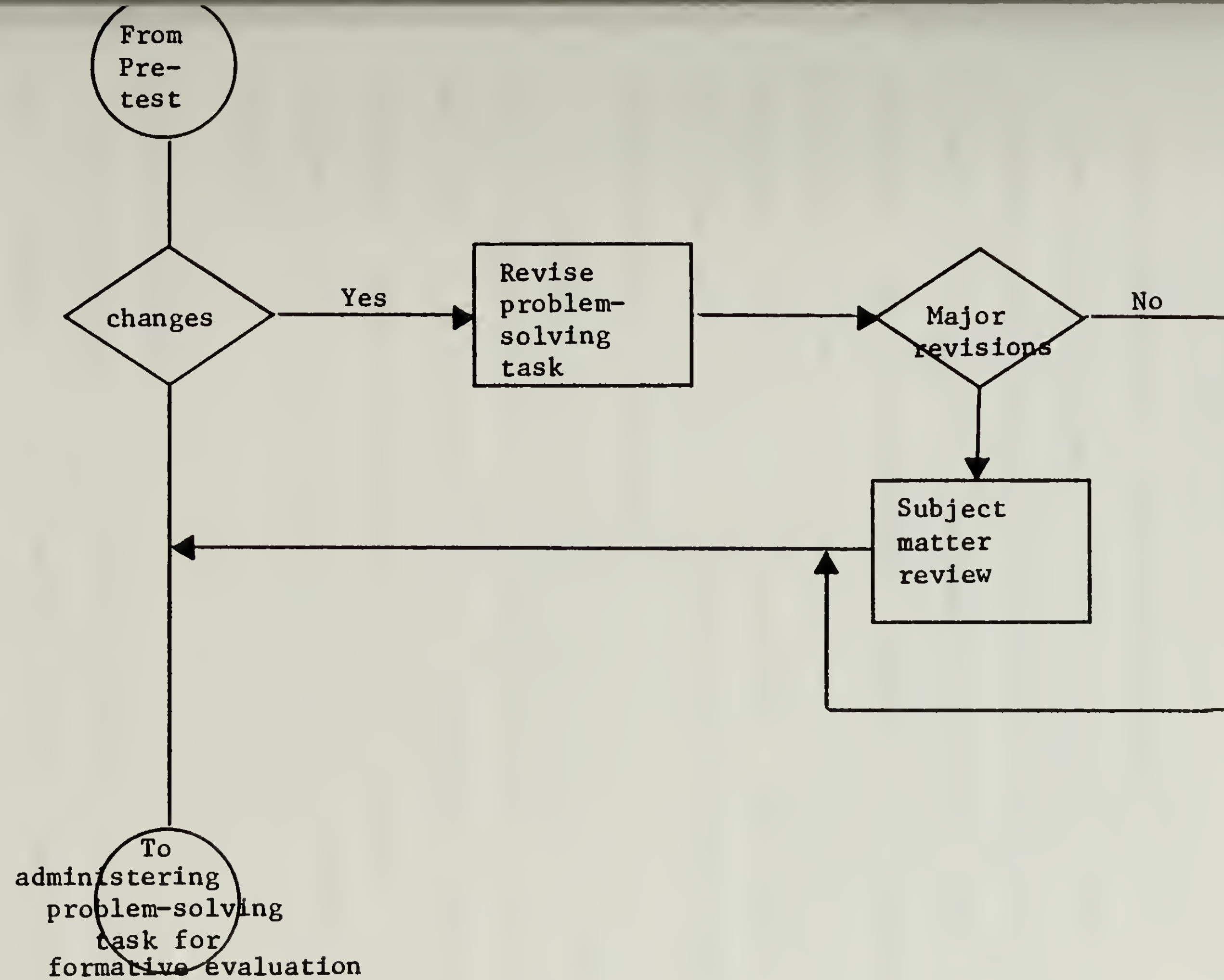


FIGURE 6. Modification of Problem-solving Task

procedural model, administration of the problem-solving task for formative evaluation.

F. ADMINISTERING OF THE PROBLEM SOLVING TASK

The sixth component of the procedural model (see figure 7) assumes that the problem-solving task has now been perfected and is ready to be used to evaluate the incorrect responses for the particular concept. A reasonably balanced population of test subjects by sex, specialization and professional levels are to be provided. The test is to be administered in the same methodology as was used in the pretesting of the instrument.

The final study should be administered to 20-30 students, but the number will vary depending on the particular concept that is being evaluated. Successful completion of this component moves the sequence of the response evaluation process to the last component of the procedural model, data analysis.

G. DATA ANALYSIS

The analysis of the results appears in the final chapters. However, a brief outline may be useful in interpreting the data analysis (see figure 8).

The first step examines the incorrect form of the responses by subject and by problem. The influence of ability level, sex and type of problem are considered next. Finally, the data will be analyzed as its applicability to CAI.

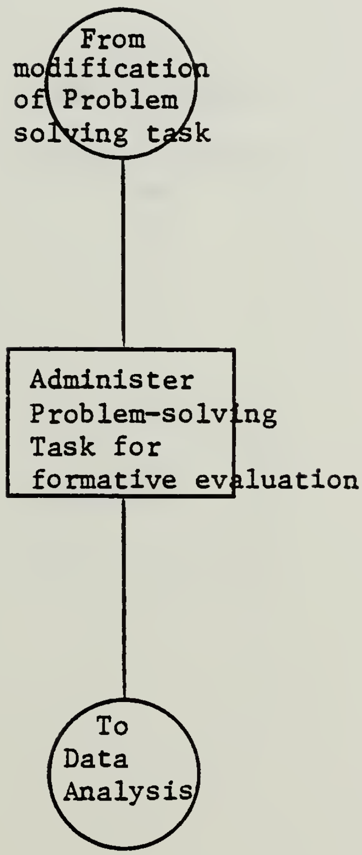


FIGURE 7. Administering Problem-Solving Task

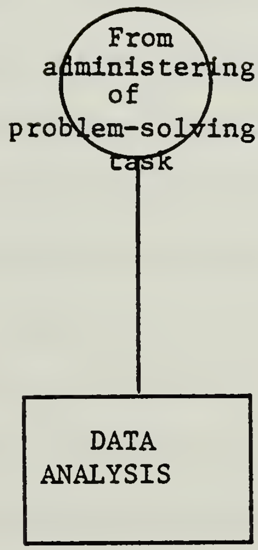


FIGURE 8. Data Analysis

IV. DATA ANALYSIS

The final study included sixty-one subjects. Thirty-seven of these were obtained from prior research (Weissinger-Baylon, 1978) and twenty-four (see Appendix A) were obtained during the course of this study. They provide a group reasonably balanced by sex, specialization, professional level, and most important, ability to understand the particular concept--the upper bound of a set.

The test group was comprised of 23% females and 77% males. Of these, 33% were graduate or undergraduate students in the mathematical sciences and 67% were undergraduates or non-mathematical science graduates. All subjects were volunteers, and the time required to do this experiment varied from 10-29 minutes per subject. The final test instrument consisted of a definition, a practice problem, and nine problem-solving sets (see figure 9).

As is true in the analysis and evaluation of any problem-solving task, the principal challenge lies in summarizing the protocol data. Using data from 671 problems obtained during this research, the response evaluation technique for the upper bound of a set concept was evaluated from three different standpoints: (a) evaluation of its ability to isolate the major incorrect responses for a particular

FINAL TASK DESCRIPTION

Set Number	Set
Definition	Let E be a subset of the real numbers, R. An element, m, of R is said to be an upper bound of E if x is equal to or less than m for every x in E
Practice Problem	Let E be the set $\{1,2,3,4\}$, i.e. the set consisting of the first four positive integers. Does E have an upper bound?
Set 2	The set of natural numbers, N (i.e., the positive integers).
Set 4	A finite set.
Set 5	$\{1, 1/2, 1/3, \ldots\}$
Set 6	The set of real numbers, R.
Set 11	The non-negative numbers
Set 13	The closed interval $[3,5]$
Set 17	$\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$
Set 18	$\{\tan x$, where $x = \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots\}$
Set 21	$\{1,2,4,8,16, \ldots\}$

Figure 9. The sequence of steps to perform the task consisted of: (1) reading the definition, (2) and then solving the practice problem. Each subject was then instructed to determine whether or not each of the sets had an upper bound. During the process of the problem solving task, subjects were asked to provide verbal and written reports of their thought processes, and draw mental pictures, if any occurred.

concept, (b) evaluation of response error statistics by type of set, and type of set between ability level and sex and,(c) evaluation of subjects response pattern. Results of each of these three evaluation efforts are reported in the following subsections.

A. ISOLATION OF MAJOR INCORRECT RESPONSES

Prior research has dictated the existence of an infinite number of theoretical correct and incorrect responses for any particular concept, depending on the nature of the behavior and the level of skill of the individual. [Taber, Glaser, Schaeffer, 1965, p. 19]. Keeping this fact in mind, the isolation of major incorrect responses becomes of the utmost importance during the final evaluation of any problem-solving task.

From the educators preliminary knowledge of the concept material and the initial analysis of the pretest results--procedural model, step two and four respectively, seven major incorrect responses or incorrect concept forms were identified as being possible for the upper bound concept. They were classified as follows:

1. Positive infinity is the upper bound for all sets.
2. Infinite number of points has no upper bound.
3. An infinite set has no upper bound.
4. The upper bound has to be a member of the set, if the set is to have an upper bound.

5. Misreading of the problem.

6. Misunderstanding the problem.

7. Other (Ignoring the problem, unable to do problem, etc.)

Analysis of the above incorrect response concepts indicated that incorrect concept forms two and three (infinite number of points has no upper bound and infinite set has no upper bound, respectively) were very similar in construct and may be combined to state that: "no upper bound exists for an infinite set." In addition, incorrect concept form six (misunderstanding the problem) may be placed in the incorrect concept form seven (other) category, since the relationship between the two concept forms were similar in nature. This initial detailed analysis of all the possible major concept forms resulted in the isolation of five incorrect concept forms for the particular upper bound concept.

Application of the five incorrect concept forms to the problem-solving sets, illustrated, theoretically, the errors that an individual would or should obtain with each particular incorrect concept form that he had learned (see figure 10). It must be noted that the final test instrument was designed and modified (procedural model, step five) so that no more than one theoretical incorrect concept applied to each problem-solving set.

THEORETICAL INCORRECT CONCEPT APPLICATION TO SETS

Set Number	SET DESCRIPTION	THEORETICAL INCORRECT CONCEPTS: Plus infinity is upper bound of everything	Infinite set has no upper bound	Upper Bound must be an Element of the set	MISREAD PROBLEM	OTHER
2	The set of natural numbers, N (i.e. the positive integers).	X			X	X
4	A finite set				X	X
5	{1, 1/2, 1/3, ...}		X		X	X
6	The set of real numbers, R.	X			X	X

Figure 10 The above sets are listed as producing errors if the particular incorrect concept is applied

THEORETICAL INCORRECT CONCEPT APPLICATION SETS (CON'T)

Set Number	SET DESCRIPTION	THEORETICAL INCORRECT CONCEPTS: Plus infinity is upper bound of everything	Infinite set has no upper bound	Upper Bound must be an Element of the set	MISREAD PROBLEM	OTHER
11	The non-negative numbers	X			X	X
13	The closed interval [3,5]		X		X	X
17	$\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$	X			X	X
18	$\{\tan x$, where $x = \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots\}$		X		X	X
21	$\{1,2,4,8,16, \ldots\}$	X			X	X

<u>Figure 10</u> The above sets are listed as producing errors if the particular incorrect concept is applied.

Using the above information, the final test results were analyzed and the errors were evaluated in regards to the five major incorrect concept forms (see figure 11). The analysis indicated that 94 percent of the errors fell into the particular error category that was determined, from the theoretical application of incorrect concept forms, to produce an error for the individual problem-solving set. In addition, 80 percent of the errors fell into just two major incorrect concept categories: plus infinity is the upper bound of everything and an infinite set has no upper bound.

It is reassuring to note that the absence of response errors for the major incorrect concept form "the upper bound must be an element of the set" indicated that the total number of incorrect response forms for this particular concept may be further reduced to only four major categories. This is a reduction of 43 percent from the number initially determined in the early stages of the response evaluation model.

B. SUBJECT AND SET TYPE EVALUATION

Results of the "subject to set evaluation" is discussed in two parts: evaluation of type of sets, and evaluation of subject, ability level and sex between type of set.

The errors for the particular sets were tabulated in figure 12. The sets are classified as being either

SUBJECT ERRORS BY INCORRECT CONCEPTS

Set Number	SET DESCRIPTION	INCORRECT CONCEPTS			MISREAD PROBLEM			
		Plus infinity is the upper Bound of everything	Infinite Set has no upper bound	Upper Bound must be an element of the set	Misread term "non-negative"	Other	Did not know Tangent Function	OTHER
2	The set of natural numbers, N (i.e. the positive integers)	T16,T30,T44 S6,S7,S9, S12						
4	A finite Set		T17 S11,S16, S20					S4
5	{1,1/2,1/3, ...}		T17,T28, T48 S17,S20, S22,S24					

Figure 11 The errors incurred by the test subjects are listed with the incorrect concepts that produced the particular error. The "T" errors were obtained from test results of a prior research project (Weissinger-Baylon, 1978, pp. 1-224 Appendix), while the "S" errors were obtained from test results in this research project, Appendix A.

SUBJECT ERRORS BY INCORRECT CONCEPTS (CON'T)

Set Number	SET DESCRIPTION	INCORRECT CONCEPTS			MISREAD PROBLEM			
		Plus infinity is the upper Bound of everything	Infinite Set has no upper bound	Upper Bound must be an element of the set	Misread term "non nega-tive"	Other	Did not know Tangent Function	OTHER
6	The set of real numbers, R.	T16,T44 S7,S9,S12						
11	The non-negative numbers	T16,T36,T44 S7,S9,S12			T19,T24 S2,S10			
13	The closed interval [3,5]		T28 S20					
17	$\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$	T16,T30,T36 T44,S6,S9				S1		
18	{ tan x, where $X=\pi, 2\pi, 3\pi, 4\pi, 5\pi \ldots\}$		T17,T23,T37 T40,S2,S3,S7, S8,S20,S21, S22, S24.				T16,T29 S15,S17 S19	T-46 S4

SUBJECT ERRORS BY INCORRECT CONCEPTS (CON'T)

Set Number	SET DESCRIPTION	INCORRECT CONCEPTS			MISREAD PROBLEM			
		Plus infinity is the upper bound of everything	Infinite Set has no upper bound	Upper Bound must be an element of the set	Misread term "non nega-tive"	Other	Did not know Tangent Function	OTHER
21	{1,2,4,8,16, ...}	T36 S6						

<u>Figure 11</u> The errors incurred by the test subjects are listed with the incorrect concepts that produced the particular error. The "T" errors were obtained from test results of a prior research project (Weissinger-Baylon, 1978, pp. 1-224 Appendix), while the "S" errors were obtained from test results in this research project, Appendix A.

SET DESCRIPTION, CATEGORIES AND ERRORS

SET NUMBER	SET DESCRIPTION	CATEGORY	ERRORS No.	ERRORS %
2	The set of natural numbers, N (i.e. the positive integers)	Abstract	7	11
4	A Finite Set	Abstract	5	8
5	$\{1, 1/2, 1/3, \ldots .\}$	Concrete	7	11
6	The set of real numbers, R	Abstract	5	8
11	The non-negative numbers	Abstract	10	16
13	The closed interval $[3,5]$	Concrete	2	3
17	$\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$	Concrete	7	11
18	$\{\tan x$, where $x=\pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi \ldots\}$	Abstract	19	30
21	$\{1,2,4,8,16, \ldots\}$	Concrete	2	3

Figure 12   The set description where presented exactly as above. Each set is classified as being concrete or abstract. Number and percentage of errors are for 61 test subjects.

"Abstract," those that may require interpretation, for example: "The non-negative numbers," and "Concrete," those which subjects can analyze directly, for example: "{1,2,4,8,16,...}." Statistical analysis indicated that the error rates for the abstract sets were considerably higher than the concrete sets (72% and 28%, respectively). The difference may be attributed to three significant factors: the final instrument consisted of five abstract sets compared to only four concrete sets; the error rate for set eighteen is abnormally high; and due to the inherent nature of the abstract type of set.

Deletion of set eighteen from the test instrument to remove the problem solving set with the abnormally high error rate and to provide an equal distribution of abstract and concrete sets, still resulted in an error rate difference between the two types of sets of 60 percent for abstract sets and 40 percent for concrete sets. Therefore, is the inherent nature of the type of sets a significant factor on the type of response? Consider for example set eleven:

Set 11. The non-negative numbers.

Ten subjects answered it incorrectly: six by incorrectly applying the upper bound definition, and four by misunderstanding the term "non-negative." Now, consider a mathematically similar task, set twenty-one:

Set 21. {1,2,4,8,16,...}.

Both sets are infinite with the only difference that set eleven is written in abstract terms while set twenty-one is written in concrete terms consisting of five elements. Since the differences between the sets are minor, error rate comparison is surprising. Indicating that the inherent nature of a problem-solving set is a significant factor in the type of response that may be obtained.

Next, error response for subject, ability level, sex and type of set (see figure 13) is analyzed and provided below:

1. It was found that the overall response error rate for the problem-solving instrument was 11.7 percent. Even though the test was designed to be easy and administered to subjects who had the ability to understand the particular concept, this indicates that individuals, as problem solvers, are often unreliable for objectively easy tasks.

2. There was a significant difference of 3.3 percent response error rate between the "mathematical science" group and the "other" group. The "mathematical science" group consisted of graduates from the Naval Postgraduate School and Stanford University and undergraduates from Colorado State University in the mathematical sciences (Mathematics, Computer Science, Statistics, and Operations Research).

SUBJECT IDENTIFIERS, SEX, FIELD, LEVEL, AND ERRORS

Subject Number		Sex	Field	MATH SCIENCE	YEAR	Errors No.	Errors %
S1	C.D.	M	CSM		GRAD	1	11
S2	L.M.	F	CS	X	GRAD	2	22
S3	T.V.	M	CSM		GRAD	1	11
S4	R.W.	M	CSM		GRAD	2	22
S5	S.L.	M	CSM		GRAD	0	0
S6	P.H.	M	CS		GRAD	3	33
S7	Y.K.	M	CSM	X	GRAD	4	44
S8	R.F.	M	D.P.		UND	1	11
S9	F.K.	M	D.P.		UND	4	44
S10	D.M.	F	MATH	X	UND	1	11
S11	S.W.	M	M.E.		UND	1	11
S12	B.N.	M	C.E.		UND	3	33
S13	E.C.	F	MATH	X	UND	0	0
S14	G.J.	M	C^3		GRAD	0	0
S15	R.C.	M	D.P.		UND	1	11
S16	K.G.	M	D.P		UND	1	11
S17	E.T.	M	D.P.	X	UND	2	22
S18	J.L.	M	C.S.		GRAD	0	0
S19	L.N.	F	C.E.		UND	1	11
S20	T.M.	M	D.P.	X	UND	4	44
S21	A.C.	M	D.P.		GRAD	1	11
S22	J.T.	M	C.S.		UND	2	22
S23	J.B.	M	M.E.		UND	0	0
S24	S.M.	M	M.E.		UND	2	22
T15	R.E.	M	O.R.	X	GRAD	0	0
T16	J.M.	M	C.S.	X	GRAD	5	55
T17	P.M.	M	M.D.		M.D.	3	33
T18	L.W.	M	MATH	X	Ph.D.	0	0
T19	N.B.	M	MATH	X	GRAD	1	1
T20	R.P.	M	C.S.	X	GRAD	0	0
T21	R.R.	M	MATH	X	GRAD	0	0
T22	J.F.	M	M.D.	X	GRAD	0	0
T23	D.D.	M	STAT	X	GRAD	0	0
T24	A.K.	M	MATH	X	GRAD	1	11

Figure 13. The error values are for nine sets.

SUBJECT IDENTIFIERS, SEX, FIELD, LEVEL, AND ERRORS

Subject Number	Sex	Field	Math Science	Year	Errors No.	%
T25 H.S.	M	MATH	X	GRAD	0	0
T26 S.K.	M	MATH	X	GRAD	0	0
T27 B.B.	M	BIOL		UND	0	0
T28 J.A.	F	Phys/CS		GRAD	2	22
T29 S.L.	F	Lang		GRAD	1	1
T30 P.W.	M	MATH	X	Ph.D.	2	22
T31 J.H.	M	MATH	X	GRAD	0	0
T32 A.S.	M	STAT	X	GRAD	0	0
T33 G.K.	M	STAT	X	GRAD	1	11
T34 R.G.	M	PSY		UND	0	0
T35 S.N.	M	MATH	X	GRAD.	0	0
T36 J.N.	M	C.E.		GRAD	3	33
T37 J.K.	M	ENG		AB	1	11
T38 R.B.	M	M.E.		UND	0	0
T39 W.P.	M	M.E.		UND	0	0
T40 D.A.	M	PSY		UND	1	1
T41 N.A.	F	EDUC		GRAD	-	-
T42 J.F.	F	C.E.		UND	0	0
T43 B.B.	M	UNDECL		GRAD	0	0
T44 B.E.	M	PHYS		UND	4	44
T45 J.R.	F	UNDECL		UND	0	0
T46 N.Z.	F	UNDECL		UND	1	11
T47 V.L.	F	UNDECL		UND	0	0
T48 L.H.	F	UNDECL		UND	1	11
T49 C.H.	F	MATH		UND	0	0
T50 J.H.	M	C.E.		UND	0	0
T51 K.T.	F	PHYS		GRAD	0	0
T52 J.G.	F	UNDECL		UND	0	0

Figure 13 (con't) The error values are for nine sets.

The "other" group consisted of Naval Postgraduate School and Stanford University graduates and Colorado State University undergraduates in curriculums other than the mathematical sciences. The response error rates for the groups were: 9.4 percent for the "mathematical sciences," and 12.7 percent for the "other" group. This significant response error difference provided a strong indication that additional years of specialized training does provide an improved response performance for technically inclined concepts.

3. The response error rate for males was 12.5 percent, for females 8.7 percent. The lower response error rate for females is very surprising, since today's society is still geared towards the higher technical training of the male counterpart. The error rate difference, however, may be explained by the unequal test population of females compared to males.

4. As indicated in the set type evaluation, the error response rate for the "Abstract" set category was found to be considerably higher than the "Concrete" set category--15.1 percent and 7.4 percent, respectively. This is a result of the inherent nature of the problem. It has been found that individuals relate better to problems which may immediately be visualized. Thereby, reducing the amount

of time required for the thought process and thus, minimize the chance for error.

5. Response error rates for subject's ability level between set type and subject's sex between set type is given in figure 14 and figure 15 respectively. The figures indicate that mathematical science students are better prepared for technical concepts provided in concrete terms, however, fail to provide a better score for abstract type problems. Moreover, the female is apt to provide better responses than their male counterpart for the specific set type categories.

The above data analysis for the different categories indicates that a wide disparity exists between and within the groups that were looked at. A split halves reliability coefficient, to determine if the nine sets constitute an acceptable instrument, was obtained by regressing total errors of even numbered subjects on the total for odd numbered. The coefficient was $r = .66$, suggesting that the problem-solving task was an acceptable instrument.

C. SUBJECTS RESPONSE PATTERN

Analysis of figures ten and eleven, indicated that a definite mastery of an incorrect concept existed for some test subjects, while for others it was not so easy to determine an error pattern (see figure 16).

	Math Sci. n = 20	Other n=41
Concrete sets	.050	.085
Abstract sets	.130	.161

Figure 14 Response error rates between ability level and set type

	Male n = 47	Female n = 14
Concrete sets	.074	.071
Abstract sets	.166	.100

Figure 15 Response error rates between sex and set type

SUBJECT RESPONSE ERROR PATTERN

SUBJECT	INCORRECT CONCEPT FORM								
	Plus infinity is the upper bound for all sets			An infinite set has no upper bound			The upper bound must be an element of the set		
	No. Theor. Errors	No. Subj. Errors	%	No. Theor. Errors	No. Subj. Errors	%	No. Theor. Errors	No. Subj. Errors	%
S2				3	1	33			
S3				3	1	33			
S6	5	3	60						
S7	5	3	60	3	1	33			
S8				3	1	33			
S9	5	4	80						
S12	5	3	60						
S17				3	1	33			
S20				3	3	100			
S21				3	1	33			
S22				3	2	66			
S24				3	2	66			
T16	5	4	80						
T17				3	2	66			
T28				3	2	66			
T30	5	2	40						
T33				3	1	33			
T36	5	3	60						
T37				3	1	33			
T40				3	1	33			
T44	5	4	80						
T48				3	1	33			
Total	40	26		45	21		0	0	
Avg	3.25	65%		1.35		47%	0	0%	

Figure 16. The number of subject errors are compared to the number of problem sets that should have theoretically produced error with the particular incorrect concept form.

Consider, for example, the major incorrect concept form "plus infinity is the upper bound for all sets." Subjects S9, T16 and T44 responded incorrectly, using the particular incorrect concept, to four out of five problem-solving sets which were theoretically designed to produce errors if that particular incorrect concept form was applied. Their test results definitely indicated a mastery of the incorrect concept form as illustrated in the following S9, T16 and T44 excerpts for set eleven:

S9: "Since plus infinity is the largest possible number, it must be the upper bound."

T16: "Bounded by continuum i.e., infinity of continuum greater than infinity of natural numbers."

T44: "Then I remember very strongly that infinity is the largest. So u.b. is infinity."

The other missed sets for the particular incorrect response concept produced a similar type of incorrect reasoning process from the three test subjects.

Now consider subject T30's response for the same incorrect concept form. T30 responded incorrectly to only two out of five problem sets. Excerpts from the missed problem sets are illustrated below:

Set 2: "Nothing can be to right of all of them, therefore, upper bound exists."

Set 17: "Square root is unbounded (i.e., off to right). Therefore, upper bound exists."

The response was implicit, but not stated as clearly. Subject T30 seemed to follow the correct process for an infinite set being unbounded, but then stated "therefore, upper bound exists."

The analysis indicated that the other errors produced by the test subjects followed a similar pattern as was shown above. With the exception of S20, no other test subject answered all of the problem sets, designed to produce an error for a particular concept, incorrectly.

It is surprising to note that an error response pattern for each subject could not be determined. Therefore, is it meaningful to even talk of error patterns? Brunner, Goodnow, and Austin (1956) discussed the role of errors and indicated that a notable flexibility of the subject's behavioral pattern exists. Different strategies will be formed for certain information. Further explanation for inconsistencies in error response patterns is provided in the following statement:

> . . . . heuristics do not guarantee any solution at all; all that can be said for a useful heuristic is that it offers solutions which are good enough most of the time.
>
> [Feigenbaum and Feldman, 1963, p. 6]

This description is consistent with the behavior of the subjects error patterns.

The error pattern for test subjects with the incorrect concept form "the upper bound must be an

element of the set" were not analyzed, since the test instrument was designed to produce no errors for that concept--making it difficult to analyze a set error pattern.

V. CONCLUSIONS AND RECOMMENDATIONS

The proposed complete response evaluation approach described here appears promising. Test results have demonstrated that the developed technique provides the educator with a method to structure the CAI course material towards responses which a student is more likely to provide and, therefore, will prove to be most effective for the individual student. As it stands, the approach is ready for use by CAI lesson writers and evaluation personnel. This final section summarizes the observed strengths and weaknesses of the proposed technique, suggests specific areas for further development, and outlines a number of recommendations for utilizing the response evaluation approach.

A. EFFECTIVENESS OF THE RESPONSE EVALUATION PROCESS AND SUGGESTIONS FOR FURTHER DEVELOPMENT

On the basis of test results discussed in the preceding sections of this report, it can be concluded that the response evaluation process for a particular concept was reasonably effective; substantially more than the current processes that it may replace. Use of this approach resulted in the dramatic reduction of the infinite number of possible theoretical incorrect responses for a particular concept to a mere three. Also, substantive, although not

striking, improvements in the methodology for administering the final instrument were discovered.

It must be admitted that, in terms of one of the more central measures of success, discovery of an response error pattern for each individual subject, is not as dramatic as might have been hoped for. A set error pattern was discovered for some individuals, but could not be generalized for all test subjects.

Two major processes of the response evaluation approach, the methodology used, and the error response pattern will now be examined and discussed in some detail.

1. Methodology Used

The imagery and verbal response methodology is sound. Although there is no proof that imagery dramatically reduces problem-solving errors (Weissinger-Baylon, 1978), it is an effective method for evaluation of thought processes for the particular application for which it was used. It must be noted, however, that some problem-solving tasks make heavy demands on imagery, while others do not. This is not the result of the inability of the subjects to produce them, but on the inherent nature of the task. While problems were encountered in two areas, they were more of the nature of management problems rather than problems with the methodology per se.

First, the test subjects were unaware of the process of using imagery in their problem-solving task and were reluctant and not ready to accept its use. Subjects had to be continually urged to put down any mental images, if and

when they occurred. This reluctance to describe mental images is not uncommon in the imagery controversy, which dates back to the early part of this century. The imagery controversy raged between psychologists with Holt (1964) and Titchener (1926) pleading for a new and more direct consideration for mental imagery, while behaviorists rejected the idea as unscientific. It was not until the mid-sixties when Shepard (1978) and others discussed the importance of imagery in the problem-solving role, that it was more readily accepted by cognitive psychologists. Therefore, in addition to the recognized newness of the use of imagery in problem-solving, the further reluctance of test subjects to use mental images to solve a problem-solving task may best be explained by Arnheim:

> images must be highly abstract since the mind operates often at high levels of abstraction. But to get at these images is not easy. I mentioned that a good deal of imagery may occur below the level of consciousness and that even if conscious, such imagery may not be noticed readily by persons unaccustomed to the awkward business of self-observation. At best, mental images are hard to describe and easily disturbed.
>
> [Arnheim, 1969, p. 116]

There is no doubt that the use of mental imagery in problem-solving can be, and hopefully will be refined through subsequent psychological studies. Once the subjects are aware of the role of mental imagery, their reluctance to report it will diminish and its management will become more efficient.

The second area concerned over-emphasizing verbal reporting versus the written reports. Since it is easier to speak than to write, some subjects initially tended to "overplay" their verbal responses, while "down-playing" the written protocol. Even though more accurate and complete, analysis and management of the taped verbal protocols became difficult when it could not be compared to the subject's written protocols. Further, to consider the opposite extreme, some subjects tended to shy away from any verbal responses when they discovered that they were to be tape recorded. Therefore, even though tape recordings are more accurate and complete and, in some cases, also provide historical reference, their value is dependent on its application and its economic feasibility.

2. <u>Error Response Pattern</u>

One of the main goals of this study was to discover a response error pattern for each subject. This discovery would have led to a more individualized package of instruction for the intended user. However the results were less than dramatic, showing irregularities and inconsistencies in subject responses. For example, even though test results for some subjects definitely indicated that they had learned an incorrect form of the concept, obtaining an error rate as high as four out of five possible errors, it was not conclusive. An important question that could be asked from these results: "if the incorrect response form was definitely learned, why were not five out of five errors produced?". Another example, showing irregularities

or "funny stuff" in subject's responses are shown in test results where the subject is successfully progressing through the problem, but then supplies the wrong response. The principal challenge, therefore, lies in analyzing the protocol data and answering the following questions:

(1) How may these inconsistencies and irregularities be explained?

(2) Where do we make the cut-off mark to determine if a subject has learned an incorrect response form: is it at fifty percent incorrect responses for a particular concept or dare we raise or lower this mark?

These are difficult questions. A possible solution to the first question may best be explained by Bruner, Goodnow and Austin:

> . . . strategies as employed by people are not fixed things. They alter with the nature of the concept being sought, with the kinds of pressures that exists in the situation, with the consequences of behavior, etc. And this is of the essence. For what is most creative about concept-attainment behavior is that the patterning of decisions does indeed reflect the demands of the situations in which the person finds himself.
>
> [Bruner, Goodnow, Austin, 1956, p. 55]

In addition, Feigenbaum (1963) pointed out that sometimes the process we use to discover or reveal something, produces incorrect responses. The interesting question is "how often"?

There have been many theories on how the human mind processes a problem-solving task. One of these, which seemed to be the most plausible, and from the protocol

data, the one generally followed in this study, stated that the problem-solving process depended upon the presence of certain previously learned rules (or in simpler cases, concepts). These previously learned rules and concepts had to be recalled, as a first step, using instructions, or in the case of this experiment--the upper bound definition. Whatever stimuli was used to recall the previously learned rules, the externally applied guidance presumably functions to increase the vividness (or "availability") of some previously learned entity.

At the next stage, the individual searched for and selected the recalled rules that were specifically relevant to the stimulus situation, and rejected those which were not. There is bound to be some "noise" at this stage of the game, and the process is one of distinguishing the noise from the relevant signals. For example, if the subject is searching for the upper bound of a set, the geometric progression of the set may be quite irrelevant. If he is searching for the trigonometric relationship for "tangent" at different values for "pi," the size of the triangle may be quite irrelevant. Conceptualization of these features of the stimulus situation may have been recalled; but they must be distinguished from other concepts and discarded.

Combining subordinate rules was the next stage of the problem solving process. There are many ways in which

recalled rules and concepts may be combined; the object is to find the correct one. For example, consider set five:

Set 5: {1, 1/2, 1/3,...}

The three dots (...) would indicate the rule that the set is infinite. However, the set also stresses consideration for the rule when the numbers are progressing in descending order. Presumably, in this stage, the rules were narrowed to a relatively few likely combinations, and thus reduced the time of search.

At the next stage, the subject arrived at a provisional rule which he believed may solve the problem. This rule was then subjected to verification by carrying out the operations it suggested with reference to the stimulus objects.

The final stage that the subject was found to perform was verification. Here, the provisional rules were checked by application to the specific problem. Should verification not work, the subject returned to the task of trying new combinations of rules. When verification did work, the solution had been acquired.

In summary, the problem solving process used may be thought of as a linear sequence of operations that are actively carried out by the subjects according to a system of links influenced by a set of rules and laws. The inconsistencies and irregularities of the subject's responses

may be a result of the breaking of these links or wrong applications of the rules. Mackworth (1965) compares the human mind to a very efficient computer, composed of a large number of programs to handle all required situations (with modifications). He states that the principal criteria of problem solving is "choosing correctly between existing programs." Incorrect selection of these programs will provide incorrect or inconsistent solutions to the problem-solving task.

The answer to the second question must be made on a judgemental decision, taking into consideration the objectives of the particular concept. The analysis has indicated, however, that if there was less than fifty percent error response rate for a particular response error form, a definite error pattern could not be determined.

In conclusion, the many unanswered questions for defining a specific error response pattern must be left for the behavioral psychologists. In this study, the attempt was made to discover a specific error pattern for a particular concept, using mental imagery as a tool. A few explanations were provided for the inconsistencies and irregularities for subject's responses, but much remains to be done in this area of response evaluation.

B. RECOMMENDATIONS FOR THE RESPONSE EVALUATION APPROACH IN THE CAI ENVIRONMENT

Both Abrahms (1971) and a report on "materials development procedures and evaluation," by the McDonnell Douglas Astronautics Company (1980), indicated that a major shortcoming in CAI development is in the area of response evaluation. They stressed a need for a method which would guide the lesson writer, by defining more sophisticated rules by which anticipated responses to constructed responses may be judged. The technique provided in this study seems to fulfill some, if not all, of these requirements.

The major conclusions drawn from this study pertain to the finding that the response evaluation approach can be used to develop CAI lessons with the individual student in mind rather than to provide an additional step in the already tedious writing of course materials per se. Thus, the intent of the recommendations outlined below is to suggest potential benefits to be derived from broader application of an instructional technology which has been shown to be reasonably effective in this context.

An obvious next step would involve further development and implementation of the response evaluation approach in the actual CAI course material development process. The area which appears particularly promising is the identification of the principal incorrect responses for a particular

concept. The educator may now define his lesson branching structure with these incorrect response forms in mind. Providing such system responses as:

(1) "Are you aware that plus infinity is not the upper bound for all sets,"

(2) "Are you aware that an infinite set may have an upper bound,"

(3) and finally, "Are you aware that the upper bound for a set does not necessarily have to be a member of the set."

With some modification and further study, the response error pattern process could be implemented to provide for a more totally structured individualized CAI system for a particular concept.

Eventually, one of the most promising areas for the response evaluation approach would be to broaden the area of CAI instruction to a more diversified field of knowledge. For example, CAI lessons could be prepared for fields which were generally considered too complex due to their psychological response processes for their mastery.

Finally, it is recommended that any implementation of the response evaluation process be coupled with the following guidelines, proposed by Colonel Joe Nastasi, if the CAI lesson is to be a success:

(1) CAI lesson writers must be senior instructors who are experienced in their subject matter areas and in the course to be developed.

(2) CAI lesson writers should be trained programmed instruction writers.

(3) Learning how to write viable and valid lessons takes considerable time, and constant turnover of authors is disastrous to satisfactory progress.

(4) CAI lesson writers must be assigned full time duty of producing lessons for the CAI program.

(5) Close and continuous liaison must be maintained between the course, other instructors, CAI lesson writers, the educational consultant and the CAI project operations.

(6) An authorized course subject block diagram, with indicated lessons, examination questions, behavioral objectives, plan of each lesson, and lesson flowcharts must be prepared prior to writing and coding lessons, and be available to insure valid CAI lesson development.

(7) The officer in charge and chief instructor of the course being developed must retain full control and supervision of the CAI lesson writers and their efforts.

(8) All CAI lessons must be very carefully reviewed from a pedagogical and student viewpoint.

[Prokop, 1976, p. 218-219]

APPENDIX A

TEST DATA FROM SUBJECTS

S1 C.D.

DEFINITION: Let E be a subset of the real numbers, R. An element, m, of R is said to be an upper bound of E if x is equal to or less than m for every x in E.

1. $E \in R$

2. $m \in R$

3. $\forall x \leq m \Rightarrow m$ = upper bound.

PRACTICE PROBLEM: Let E be the set $\{1, 2, 3, 4\}$, i.e. the set consisting of the first four positive integers. Does E have an upper bound?

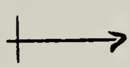
1. See four finite elements.

2. Looked for greatest value.

3. Yes, has upper bound of 4.

SET 2: The set of natural numbers, N (i.e. the positive integers)

1. I see an infinite set.



2. An infinite set is unbounded.

3. No.

SET 4: A finite set.

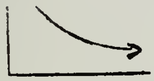
1. No mental image at first. Thought about it.
   Then saw 100 tanks.

   1 2 3 100
   • • • ... •

2. Yes, there is upper bound of greatest element in set.

SET 5: 1, 1/2, 1/3, ...

1. I see infinitely smaller numbers.



2. Therefore upper bound must be the greatest number in the set.

SET 6: The set of real numbers, R.

1. No.

2. An infinite set ($\Rightarrow$ unbounded).

SET 11: The non-negative numbers.

1. Infinite set starting at 0.

2. No, same as set 6.

SET 13: The closed interval $[3, 5]$.

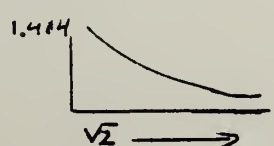
1. Yes.

2. The largest element in set is upper bound.

3. Finite set.

SET 17: $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$

1. Yes.



2. after test

No.

SET 18: $\{\text{Tan } x, \text{ where } x = \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots\}$

1. Yes. (guess since tangent is bounded by 0 , 1).

2. I see an set of 0's and 1's

$$\{0, 1, 0, 1, \ldots\}$$

SET 21; $\{1, 2, 4, 8, 16, \ldots\}$

1. I see numbers increasing.

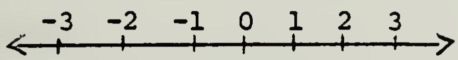
2. No, infinite set.

S2 L.M.

DEFINITION: Let E be a subset of the real numbers, R. An element, m, of R is said to be an upper bound of E if x is equal to or less than m for every x in E.

1. Definition thought:

The real numbers ordered on a number line.



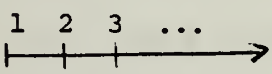
PRACTICE PROBLEM: Let E be the set {1, 2, 3, 4}, i.e. the set consisting of the first positive integers. Does E have an upper bound?

1.

2. upper bound = 4.

SET 2: The set of natural numbers, N (i.e. the positive integers)

1.



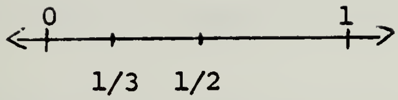
2. No bound, it goes to infinity.

SET 4: A finite set.

1. Points on the line -- bounded by the element which falls farthest to the right on the number line (like definition).

SET 5: {1, 1/2, 1/3, ...}

1. I see an infinite number of points getting smaller.

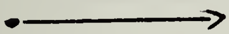
2. 

3. bounded by 0.

4. I remember definition and it asks for upper bound, therefore $\Longrightarrow$ 1.

SET 6: The set of real numbers, R.

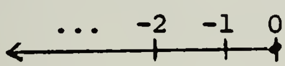
1. no bound.

2. 

SET 11: The non-negative numbers.

1. I see an infinite set.

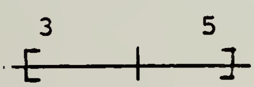
2. Upper bound $\cancel{0}$ (the first non-negative number).

3. 

4. [No lower bound]

SET 13: The closed interval $[3, 5]$.

1. Saw the largest number 5.

2. 

3. bound 5.

SET 17: $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$

1. No bound -- each number bigger than the previous number.

<u>SET 18</u>: $\{\text{Tan } x, \text{ where } x = \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots\}$.

1. I could not remember definition of tan.

2. I see ... , therefore goes to infinity.

3. No bound.

<u>SET 21</u>: $\{1, 2, 4, 8, 16, \ldots\}$.

1. Increasing infinite set.

$$\{1 \quad 2 \quad 4 \quad 8 \ldots \rightarrow$$

2. no bound.

S3 T.V.

DEFINITION: Let E be a subset of the real numbers, R. An element, m, of R is said to be an upper bound of E if x is equal to or less than m for every x in E.

1. $x \leq m$ for every x in E.

PRACTICE PROBLEM: Let E be the set $\{1, 2, 3, 4\}$, i.e. the set consisting of the first four positive integers. Does E have an upper bound?

1. Yes, 4.

2. Four is the largest number and so all of the others are less than or equal to it so it must be the upper bound.

SET 2: The set of natural numbers, N (i.e. the positive integers).

1. No upper bound → infinite.

2. Imagery-- (1, 2, 3, ...)

SET 4: A finite set.

1. I see a certain number of points.

2. Imagery -- $\{1, 2, 3, 4\}$

3. Upper bound equals largest number in set.

SET 5: $\{1, 1/2, 1/3, \ldots\}$

1. 1 is upper bound.

2. I visualize the numbers trailing off to right so 1 is largest. My first impression was that it was an infinite set with no upper bound.

<u>SET 6</u>: The set of real numbers, R.

1. Infinite set, no upper bound.

2. Imagery $\{1, 2, 3, 4, \ldots\}$.

<u>SET 11</u>: The non-negative numbers.

1. Infinite set.

2. no upper bound (see above).

<u>SET 13</u>: The closed interval $[3, 5]$.

1. 5 is the upper bound.

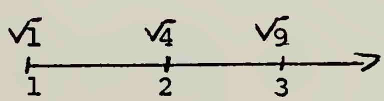
2. It is the largest number in a finite set.

<u>SET 17</u>: $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$

1. Infinite set -- no upper bound.

2. I first look at the values of the perfect squares.

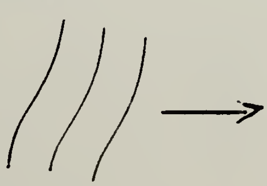
3. Imagery



<u>SET 18</u>: $\{\text{Tan } x, \text{ where } x = \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots\}$

1. Visualize coordinate plot of trig values.

2. $\tan = \dfrac{\sin}{\cos}$



3. Infinite set -- no upper bounds.

<u>SET 21</u>: $\{1, 2, 4, 8, 16, \ldots\}$

1. Infinite set no upper bound.
2. Imagery = X^2

S4 R.W.

<u>DEFINITION</u>: Let E be a subset of the real numbers, R. An element, m, of R is said to be an upper bound of E if x is equal to or less than m for every x in E.

1. DEF: For all answers ∞ is <u>no</u> bound

<u>PRACTICE PROBLEM</u>: Let E be the set $\{1, 2, 3, 4\}$, i.e. the set consisting of the first four positive integers. Does E have an upper bound?

1. E has an upper bound.

2. m = 4, because set E has ascending order.

3. 1 2 3 4

<u>SET 2</u>: The set of natural numbers, N (i.e. the positive integers).

1. 1 2 3 4 ... $+\infty$

2. There is no upper bound, unlimited.

<u>SET 4</u>: A finite set.

1. A finite set not necessarily has an upper bound, because "finite" is concerned with the number of elements in the set.

2. After asking examiner -- this set is bounded above.

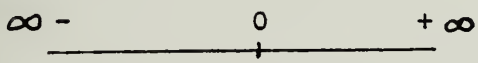
<u>SET 5</u>: $\{1, 1/2, 1/3, \ldots\}$

1. 1 0

2. The upper bound of this set is 1, because the set has a descending order in its elements.

3. m = 1

<u>SET 6</u>: The set of real numbers, R.

1. 

2. The set of real numbers has no upper bound because they go to +

<u>SET 11</u>: The non-negative numbers.

1. 

2. No upper limit, goes to ∞

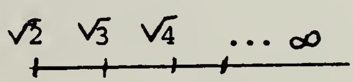
<u>SET 13</u>: The closed interval $[3, 5]$.

1. $[3, 5]$

2. The closed interval has an upper bound.

3. m = 5.

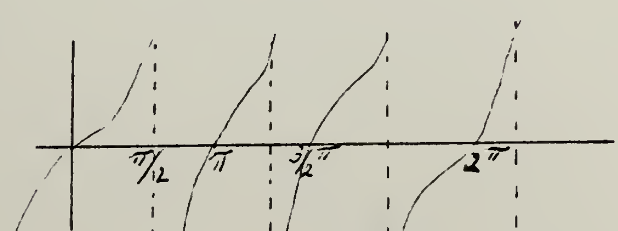
<u>SET 17</u>: $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$

1. 

2. no upper bound

<u>SET 18</u>: $\{$ Tan x, where $x = \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots\}$

1.



2. Set contains all zeroes, E $\{0, 0, 0, \ldots\}$

3. Set has no upper bound.

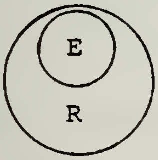
<u>SET 21</u>: $\{1, 2, 4, 8, 16, \ldots\}$

1. Set has no upper bound.

2. Set of the squares of natural numbers $\longrightarrow \infty$

S5 S.L.

DEFINITION: Let E be a subset of the real numbers, R. An element, m of R is said to be an upper bound of E if x is equal to or less than m for every x in E.

1. 

2. $m \geq [E]$

3. $x = [E] \leq m$

4. m = element in E.

PRACTICE PROBLEM: Let E be the set {1, 2, 3, 4}, i.e. the set consisting of the first four positive integers. Does E have an upper bound?

1. $[E] = (1, 2, 3, 4)$

2. m = 4

SET 2: The set of natural numbers, N (i.e. the positive integers).

1. Natural numbers, N, $\longrightarrow$ unlimited positive number.

2. So there should not be any upper bound.

SET 4: A finite set.

1. A finite set $\longrightarrow$ the set is finite.

2. So should be an upper bound of the subset E.

SET 5: { 1, 1/2, 1/3, ...}

1. (1, 1/2, 1/3, ...) $\longrightarrow$ unlimited.

2. Fraction $\longrightarrow$ 1/n.

3. 1 = the upper bound.

SET 6: The set of real numbers, R.

1. Undefined property $\longrightarrow$

2. The set of real numbers, R, $\longrightarrow$ if it means all real numbers $\longrightarrow$ then there is no upper bound.

3. If it starts from a certain number downward $\longrightarrow$ there is an upper limit.

SET 11: The non-negative numbers.

1. All non-negative numbers.

2. No upper bound except

SET 13: The closed interval $[3, 5]$.

1. 

2. 5 = upper bound.

SET 17: $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$

1. $(\sqrt{2}, \sqrt{3}, \ldots, \sqrt{7}, \ldots) \Longrightarrow \sqrt{n}$

2. $\longrightarrow$ no upper bound.

SET 18: $\{\text{Tan } x, \text{ where } x = \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots\}$

1.

$\tan \pi = \tan 2\pi$

2. upper bound = 0

<u>SET 21</u>: $\{1, 2, 4, 8, 16, \ldots\}$

1. See an infinite increasing set.

2. no upper bound $\Longrightarrow 2^m$.

S6 P.H.

<u>DEFINITION</u>: Let E be a subset of the real numbers, R. An element, m, of R is said to be an upper bound of E if x is equal to or less than m for every x in E.

1. $m \in R$.

<u>PRACTICE PROBLEM</u>: Let E be the set $\{1, 2, 3, 4\}$, i.e. the set consisting of the first four positive integers. Does E have an upper bound?

1. Yes

2. upper bound $\geq$ 4.

<u>SET 2</u>: The set of natural numbers, N (i.e. the positive integers).

1. $N = \{1, 2, 3, \ldots\}$

2. Yes, $\longrightarrow$ always a real number that greater or equal to the largest integer.

<u>SET 4</u>: A finite set.

1. $A = \{a1, a2, \ldots, ai\}$

   where i is finite number

2. yes bounded.

<u>SET 5</u>: $\{1, 1/2, 1/3, \ldots\}$

1. The set decreases.

2. Yes, bounded = 1.

SET 6: The set of real numbers, R.

1. $\longleftrightarrow$

2. No, real numbers not bounded upper side.

SET 11: The non-negative numbers.

1. No

2. Non-negative numbers could be real $\longrightarrow$ so the same as set 6 above.

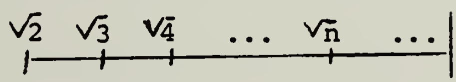
SET 13: The closed interval $[3, 5]$

1. 

2. Yes, bound ≥ 5.

SET 17: $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$

1. Yes, because the number under square root are integers; we always could find a real number that is larger than that.

2. 

SET 18: $\{$ Tan x, where $x = \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots\}$

1. 

2. always $\tan x = 0$

3. yes, an upper bound

SET 21: $\{1, 2, 4, 8, 16, \ldots\}$

1. Set of integers.

2. Yes would be upper bound.

3. Same as set 2.

S7 Y.K.

DEFINITION: Let E be a subset of the real number, R. An element, m, of R is said to be an upper bound of E if x is equal to or less than m for every x in E.

PRACTICE PROBLEM: Let E be the set $\{1, 2, 3, 4\}$, i.e. the set consisting of the first four positive integers. Does E have an upper bound?

1. Yes, the upper bound is 4.

2. Each number in the set is less than 4.

SET 2: The set of natural numbers, N (i.e. the positive integers).

1. Yes

2. There is a number greater among the other numbers.

3. Examiner questions answer:

4. Infinity is the bound.

SET 4: A finite set.

1. Yes, same as set 2.

SET 5: $\{1, 1/2, 1/3, \ldots\}$

1. yes

2. The upper bound is 1

3. Each number in the set is less than 1.

SET 6: The set of real numbers, R.

1. Yes, same as set 2.

<u>SET 11</u>: The non-negative numbers.

1. Yes, same as set 2.

<u>SET 13</u>: The closed interval $[3, 5]$.

1. Yes.
2. 5 is the upper bound.

<u>SET 17</u>: $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$

1. No, because it is infinite.

<u>SET 18</u>: $\{\text{Tan } x, \text{ where } x = \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots\}$

1. No upper bound
2. Goes to infinity

<u>SET 21</u>: $\{1, 2, 4, 8, 16, \ldots\}$

1. No because it is infinite.

S8 R.F.

DEFINITION: Let E be a subset of the real numbers, R. An element, m, of R is said to be an upper bound of E if x is equal to or less than m for every x in E.

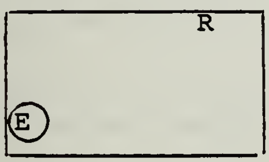
1. $\{R\}$ E

2. $\{\ldots m\}$

   subset E of real numbers, R.

3. $x \leq m$

4. m is only an upper bound of subset E if $x \leq m$

5. 

PRACTICE PROBLEM: Let E be the set $\{1, 2, 3, 4\}$, i.e. the set consisting of the first four positive integers. Does E have an upper bound?

1. E has an upper bound (4)

2. Reason is that it is a finite set, so the highest number is the upper bound.

3. $E = \{1, 2, 3, 4\}$

SET 2: The set of natural numbers, N (i.e. the positive integers)

1. $\{1, 2, 3, \ldots, n\}$

2. infinite set

3. no upper bound

SET 4: A finite set

1. $\{1, 2, 3, 4, 5\}$

2. has upper bound -- in my ex. upper bound is 5.

SET 5: $\{1, 1/2, 1/3, \ldots\}$

1. decreasing set to infinity.

2. def. of upper bound means the highest number in that set.

3. The upper bound, in ex., is the highest number (1).

SET 6: The set of real numbers, R.

1. Does not specify what R contains.

2. It is an infinite set.

3. Correction -- it does specify what R contains -- all real numbers.

4. All real numbers is an infinite set.

5. Thus no upper bound.

SET 11: The non-negative numbers.

1. Means all positive numbers.

2. All positive numbers is an infinite set.

3. No upper bound.

SET 13: The closed interval $[3, 5]$

1. Contains only numbers 3 and 5.

2. Finite set

3. Upper bound is 5.

SET 17: $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$

1. Increasing set.

2. Increasing by one square root digit.

3. In order, so that the next number will be

   $\sqrt{8}, \sqrt{9}, \sqrt{10}, \ldots \sqrt{N}\}$

4. It is an infinite set.

5. Does not have an upper bound, except for the $\sqrt{n}$, which is undefined at present.

6. I would say this set does not have an upper bound.

SET 18: $\{\text{Tan } x, \text{ where } x = \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots\}$

1. Increasing set.

2. Same as problem set 17.

3. No upper bound.

SET 21: $\{1, 2, 4, 8, 16, \ldots\}$

1. increasing set in sequence of 2^x

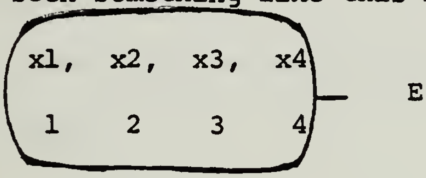
2. infinite set.

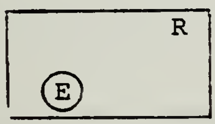
3. No upper bound.

S9 F.K.

DEFINITION: Let E be a subset of the real numbers, R. An element, m, of R is said to be an upper bound of E if x is equal to or less than m for every x in E.

1. I've seen something like this before.

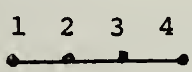
2. 

3. 

4. $xi \leq m$.

PRACTICE PROBLEM: Let E be the set $\{1, 2, 3, 4,\}$, i.e. the set consisting of the first four positive integers. Does E have an upper bound?

1. I see a finite set.

2. 

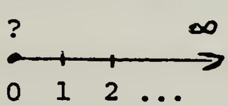
3. According to definition $xi \leq m$

4. Yes, upper bound = 4 or greater.

SET 2: The set of natural numbers, N (i.e. the positive integers).

1. what is a natural number -- is 0 a natural number?

2. This set goes to infinity



3. Since infinity is the largest number -- the upper bound is infinity.

SET 4: A finite set.

1. A$\{1, 2, 3, 4, 5\}$
2. This is just like the practice problem.
3. Upper bound.

SET 5: $\{1, 1/2, 1/3, ...\}$

1. I see the set decreasing.

   $1/4, 1/5, 1/6 \longrightarrow 1/n$

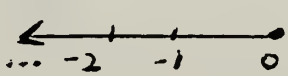
3. Since it is decreasing the largest number must be the upper bound.
4. Yes, upper bound = 1.

SET 6: The set of real numbers, R.

1. what is a real number?
2. Rational numbers such as -3/4, 18/29, 3, -6
3. Irrational numbers such as $\sqrt{2}$ $\sqrt[3]{5}$, $\sqrt[5]{-3}$
4. It (real) goes to infinity.
5. Infinity must be the upper bound, since there is no number greater than infinity.

SET 11: The non-negative numbers.

1. I see a line going towards the left to $-\infty$



2. Therefore zero must be the upper bound.

3. Wait, it wants non-negative numbers.

4. The line goes to the right to $+\infty$

5. Since + is the largest possible number, it must be the upper bound.

<u>SET 13</u>: The closed interval $[3, 5]$

1. I see a line like this

2. From previous experience this is a finite set, with boundaries 3 and 5

3. upper bound of 5.

<u>SET 17</u>: $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$

1. I first see an infinite set.

...

2. I then look at the "$\sqrt{\ }$"

3. I ignore this, since infinity suggests an upper bound.

4. Yes, upper bounded above.

<u>SET 18</u>: $\{$ Tan x, wherex $= \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots\}$

1. I see an infinite set.

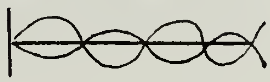
2. I immediately say, yes bounded above.

3. I look at what tangent means

$$\tan = \frac{\sin}{\cos}$$

4. what are the values for tan at π, 2π, etc.

5. I draw sine curve and cosine curve.



6. Determined that tan x = $\{0, 0, 0, \ldots\}$

7. Upper bound of 0 or greater.

<u>SET 21</u>: $\{1, 2, 4, 8, 16, \ldots\}$

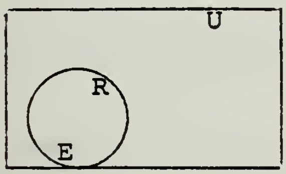
1. I see a progression -- 2^x

2. It goes on -- no upper bound.

S10 D.M.

DEFINITION: Let E be a subset of the real numbers, R. An element, m, of R is said to be an upper bound of E if x is equal to or less than m for every x in E.

1.

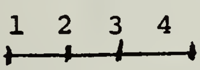


2. $x \leq m \Longleftrightarrow R$

PRACTICE PROBLEM: Let E be the set $\{1, 2, 3, 4\}$, i.e. the set consisting of the first four positive integers. Does E have an upper bound?

1. This problem is obvious.

2.



3. it is finite, therefore upper bound of 4.

SET 2: The set of natural numbers, N (i.e. the positive integers)

1. I immediately see an infinite set going towards plus infinity.

2. This implies to me that no upper bound exists.

SET 4: A finite set.

1. Obvious, no upper bound.

SET 5: $\{1, 1/2, 1/3, \ldots\}$

1. I see an infinite set.

2. It decreases to zero $\rightarrow$ therefore, the largest number must be the upper bound.

3. Almost tricked me by showing infinite set.

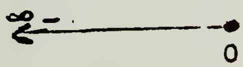
<u>SET 6</u>: The set of real numbers, R.

1. I immediately see an infinite set, which causes me to say no upper bound.

2. I see no other images.

<u>SET 11</u>: The non-negative numbers.

1. I draw a picture



2. Since 0 is the first non-negative number, it must be the upper bound.

<u>SET 13</u>: The closed interval $[3, 5]$

1. There are an infinite number of points between 3 and 5, but endpoints are 3 and 5.

2. Yes, upper bound.

<u>SET 17</u>: $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$

1. I see an infinite set.

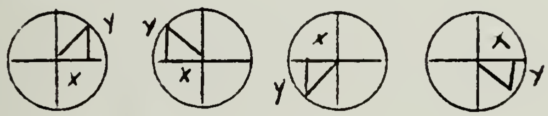
2. I ignore the "$\sqrt{}$"

3. Set goes to infinity

4. No upper bound

<u>SET 18</u>: $\{$ Tan x, where $x = \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots\}$

1. Tan x = x/y

2. 

3. whenever x = 0, y is either 1 or -1

4. therefore tan x = $\{0, 0, 0, \ldots\}$

5. Set has an upper bound

<u>SET 21</u>. $\{1, 2, 4, 8, 16, \ldots\}$

1. I see an infinite set.

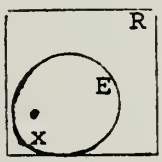
2. Progression = 2^n

S11 S.W.

DEFINITION: Let E be a subset of the real numbers, R. An element, m, of R is said to be an upper bound of E if x is equal to or less than m for every x in E.

1. $x \rightarrow E \rightarrow R$

2.

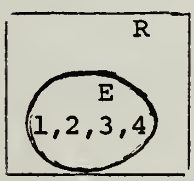


3. $m \rightarrow R$

4. $x \leq m$

PRACTICE PROBLEM: Let E be the set $\{1, 2, 3, 4\}$, i.e. the set consisting of the first four positive integers. Does E have an upper bound?

1.



2. Since E is a finite set, there exists a R greater than E.

3. Upper bound.

SET 2: The set of natural numbers, N (i.e. the positive integers).

1. No real images.

2. Since natural numbers go on indefinitily, it implies an infinite set.

3. No upper bound.

<u>SET 4</u>: A finite set.

1. How can I assign an m?

2. I don't see any numbers or elements.

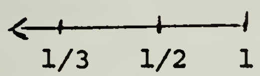
<u>SET 5</u>: $\{1, 1/2, 1/3, \ldots\}$

1. I see

   ...

2. Infinite set has no bound, but wait it is decreasing.

3. I draw picture



4. The progression is 1/n $\Rightarrow$ goes to 0

5. Upper bound is the largest number -- 1.

<u>SET 6</u>: The set of real numbers, R.

1. I see this immediately as an infinite set.
   Therefore not bounded.

2. No mental pictures appeared.

<u>SET 11</u>: The non-negative numbers.

1. Draw picture of the non-negative numbers.

2. Goes to infinity.

3. No upper bound.

SET 13: The closed interval $[3, 5]$

1. First looked at bracketed numbers and recognized m=5.

2. Next looked at the word interval this implied infinite set.

3. Since it is bracketed $m \geq 5$.

SET 17: $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$

1. Saw numbers with square root signs.

2. Saw dots which implied infinity and no way of assigning an m.

3. numbers are increasing under square root signs.

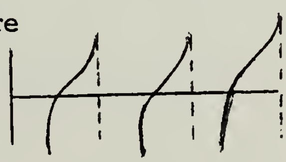
4. infinite set, m can not be defined.

SET 18: $\{$ Tan x, where x $= \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots\}$

1. First saw an infinite set.

2. The value π for tan looks familiar.

3. Draw picture



4. $\tan x = \dfrac{\sin x}{\cos x} \longrightarrow \pi = 0 \quad 2\pi = 0$

5. the set is $\{0, 0, 0, \ldots\}$.

6. upper bound.

SET 21: $\{1, 2, 4, 8, 16, \ldots\}$

1. infinite set came to mind first.

2. Same as set 17.

S12 B.N.

DEFINITION: Let E be a subset of the real numbers, R. An element, m, of R is said to be an upper bound of E if x is equal to or less than m for every x in E.

1. First time I read definition it didn't make sense.

2. $E \in R$

3. $m \in R$

4. $x \in E$

5. If all of the above true, then

$$x \leq m$$

for upper bound to be true.

6. Checking to see if that makes sense.

PRACTICE PROBLEM: Let E be the set $\{1, 2, 3, 4,\}$, i.e. the set consisting of the first four positive integers. Does E have an upper bound?

1. Instantaneous reaction.

2. Verify upper bound, also lower bound.

3. Yes, finite and bounded in both direction

SET 2: The set of natural numbers, N (i.e. the positive integers).

1. I see an infinite line going to the right.

2. I remember definition said real numbers

3. Since infinity of real numbers not same as infinity of natural numbers.

4. The set is bounded by infinity of real numbers.

<u>SET 4</u>: A finite set.

1. It has bound (from memory, practice problem).

<u>SET 5</u>: $\{ 1, 1/2, 1/3, \ldots \}$

1. see infinite set.
2. inspect
3. Find pattern $\longrightarrow$ 1/n.
4. Get smaller to right.
5. $\therefore$ Find largest number.
6. Bounded by 1.

<u>SET 6</u>: The set of real numbers, R.

1. This is just like set 2.
2. Is there a number $\geq$ infinity for real.
3. No, $\therefore$ bounded by infinity.

<u>SET 11</u>: The non-negative numbers.

1. These are natural numbers.
2. This type of question was already asked.
3. Is this a trick??
4. It goes to infinity $\therefore$ bounded by infinity of real.

SET 13: The closed interval $[3, 5]$

1. I see 3 and 5.
2. 5 is largest.
3. upper bound.

SET 17: $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$

1. This is an infinite set from dots.
2. Sequence $\longrightarrow \sqrt{n}$
3. Can I find m larger than $\sqrt{n}$
4. I can't see m
5. not bounded.

SET 18: $\{$ Tan x, where $x = \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots\}$

1. Compute tangent

$$\tan x = \frac{\sin x}{\cos x}$$

   Put in the π's for the x's

2. $\cos \pi$ 1 or -1; $\sin \pi \rightarrow 0$
3. Constant 0.
4. $m \geq 0$ $\therefore$ bounded.

SET 21: $\{1, 2, 4, 8, 16, \ldots\}$

1. Numbers seem to be increasing in some order.

2. I think N^2

3. See dots $\therefore$ infinity

4. Can I find m?

5. No.

S13 E.C.

DEFINITION: Let E be a subset of the real numbers, R. An element, m, of R is said to be an upper bound of E if x is equal to or less than m for every x in E.

1. Definition is familiar.

2. $\forall x,\ x \in E,\ x \leq m$

3. $E \subset R$

4. If true, upper bound.

PRACTICE PROBLEM: Let E be the set $\{1, 2, 3, 4\}$, i.e. the set consisting of the first four positive integers. Does E have an upper bound?

1. finite set

2. $x \leq m$

   $x = 1, 2, 3, 4$

3. $m \geq 4$

4. Bounded

SET 2: The set of natural numbers, N (i.e. the positive integers).

1. Natural numbers do not include irrational numbers

2. Horizontal in space.

3. Unbounded to right.

4. Bounded on left.

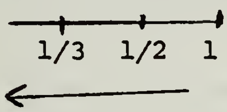
SET 4: A finite set.

1. Biggest element (automatic)

SET 5: $\{1, 1/2, 1/3, \ldots\}$

1. 1/n first thought.

2. infinite decreasing



3. If n very large sequence goes to 0.

4. 1 biggest.

SET 6: The set of real numbers, R.

1. line appeared $\longrightarrow$ continuous

2. Continuous $\Rightarrow$ infinity $\therefore$ no upper or lower bound.

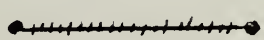
SET 11: The non-negative numbers.

1. 0, 1, 2, 3, ... $\longrightarrow$

2. Same as above: continuous $\Rightarrow$ infinity $\therefore$ no upper but has lower bound.

SET 13: The closed interval $[3, 5]$.

1. interval $\Rightarrow$ infinite number of points



2. biggest is 5 $\therefore$ m $\geq$ 5

SET 17: $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$

1. Noticed integers as written.

2. Integers unbounded.

3. Replaced $\sqrt{}$

4. Still increasing.

5. Above set unbounded.

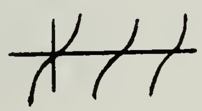
SET 18: $\{$ Tan x, where x $= \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots\}$

1. $\tan \pi = 0$

   $\tan 2\pi = 0$

2. Are all elements the same?

3. Look at graph vs. computed values



4. The elements are the same

5. must be bounded ≥ 0

SET 21: $\{1, 2, 4, 8, 16, \ldots\}$

1. infinite set

2. Progression 2^n has no real value.

3. unbounded.

S14 G.J.

DEFINITION: Let E be a subset of the real numbers, R. An element, m, of R is said to be an upper bound of E if x is equal to or less than m for every x in E.

1. What are the relationships between (x, m, E, R)?

2. $E \subset R$

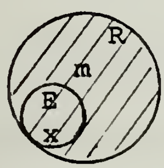
   $x \in E$

   $m \in R$

   $x \leq m$

3. I had to read definition several times.

4.



PRACTICE PROBLEM: Let E be the set $\{1, 2, 3, 4\}$, i.e. the set consisting of the first four positive integers. Does E have an upper bound?

1. Yes, the upper bound is 4.

   This is an immediate reaction, since the set is not infinite.

SET 2: The set of natural numbers, N (i.e. the positive integers).

1. No.

2. I picture mental image of above set, but continuing

   1, 2, 3, 4, ...

3. Can't be upper bound $\longleftrightarrow$ infinite.

S14 G.J.

SET 4: A finite set.

1. What is the definition of a finite set?

2. Definition states that an upper bound and lower bound exists.

images:
1, 2, 3, 4

3. Therefore has bound.

SET 5: $\{1, 1/2, 1/3, \ldots\}$

step 1. I see this set as a decreasing fraction, therefore I tentatively conclude that the values will continue to decrease.

step 2. Notice a pattern in the set. Numerator is constant, denominator is increased by 1.

1. Discovered that set $\rightarrow 0$

2. This must be the lower bound.

3. Examine set for greatest value.

4. m = 1.

SET 6: The set of real numbers, R.

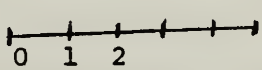
1. Immediately, I remember the definition.

2. Conclude that there is no upper bound.

SET 11: The non-negative numbers.

1. Define non-negative

image;



2. This has same property as the set of natural numbers.

3. State the same conclusion that there is no upper bound.

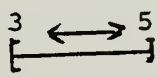
SET 13: The closed interval $[3, 5]$

1. Examine the interval.

2. State its properties: $3 \leq x \leq 5$

3. State conclusion: $m = 5$

Image:



SET 17: $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$

1. Change squares into familiar digits

$\sqrt{2} = 1.414 \quad \sqrt{3} = 1.732 \quad \sqrt{4} = 2$ etc.

2. I notice the values increasing without bound.

3. Out of habit, there is no upper bound.

4. I was busy with the calculations so I did not see any images.

SET 18: $\{\text{Tan } x, \text{ where } x = \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots\}$

1. Define function in more familiar terms.

$\tan x = \sin x / \cos x = 0$

2. Above expres-sion = 0 always, since we deal with angles where numerator is always 0.

3. Conclude that the function neither increases or decreases.

4. The upper and lower bound = 0.

<u>SET 21</u>: $\{ 1, 2, 4, 8, 16, \ldots \}$

1. Notice the numbers increase initially.

2. Confirm that this will always increase without bound.

3. Conclude that there is no upper bound.

   No imagery. Strictly computational.

S15 R.C.

DEFINITION: Let E be a subset of the real numbers, R. An element, m, of R is said to be an upper bound of E if x is equal to or less than m for every x in R.

1. m must be to the right of E (the elements of E).

2. Note that $x \leq m$

PRACTICE PROBLEM: Let E be the set $\{1, 2, 3, 4\}$, i.e. the set consisting of the first four integers. Does E have an upper bound?

1. Yes.

2. m = 4

3. Re-read definition to see if upper bound could be equal to an element of E.

SET 2: The set of natural numbers, N (i.e. the positive integers).

1. No.

   I realize set was infinite.

SET 4. A finite set.

1. Yes m = largest element.

2. I visualize a finite set as a set of points

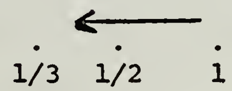
3. Took largest point as the upper bound.

<u>SET 5</u>: $\{1, 1/2, 1/3, \ldots\}$

yes, m = 1.

Visualization of 1 and 1/2 and 1/3.

Realized points were going off to the left.



Since we are interested in the values on the right

m = 1.

<u>SET 6</u>: The set of real numbers, R.

1. No.

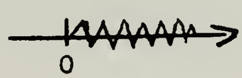
2. Automatically realized set had no upper bound.

   No image of R.

<u>SET 11</u>: The non-negative numbers.

1. No.

2. Image of right half of real number line came to mind to make sure we weren't dealing with negative numbers.



3. The set is infinite to the right.

<u>SET 13</u>: The closed interval $[3, 5]$

1. Yes m=5, no image

2. I just saw two numbers.

3 5

3. Knew answer as soon as I saw it.

SET 17: $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$

1. No, realized set was infinite

2. Did not look at $\sqrt{}$

SET 18: $\{\text{Tan } x, \text{ where } x = \pi, 2\pi, 3\pi\ 4\pi\ 5\pi\ 6\pi \ldots\}$

1. Oh no -- not a trignometric function.

2. What does tangent mean?

3. how does it relate to sin and cos?

$$\tan = \frac{\sin}{\cos} \qquad \text{or } \tan = \frac{\cos}{\sin}$$

4. I see an infinite set, but may be a trick question.

5. I will guess unbounded.

SET 21: $\{1, 2, 4, 8, 16, \ldots\}$.

No, realized set was increasing.

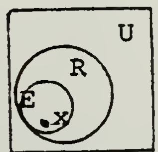
S16 K.G.

DEFINITION: Let E be a subset of the real numbers, R. An element, m, of R is said to be an upper bound of E if x is equal to or less than m for every x in E.

1. Had to read the definition a number of times before I even had the vagous idea of what an upper bound was.

2. I could not determine relationship between x and m.

3. Remembered to use Venn diagrams.



4. I now saw that m had to be in R and had to be greater or equal to x.

PRACTICE PROBLEM: Let E be the set $\{1, 2, 3, 4\}$, i.e. the set consisting of the first four positive integers. Does E have an upper bound?

1. This is not type of problems I expected, although I should have from definition.

2. It seems obvious.

3. I know 5 or greater is an upper bound, but is 4?

4. Yes.

SET 2: The set of natural numbers, N (i.e. the positive integers)

1. Read problem

2. I believe that no upper bound exists because the numbers go to infinity.

3. I remember from a previous course the conflict about 0 being in the natural numbers.

SET 4: A finite set.

1. I cannot see an m in the set.
2. I re-read the definition.
3. Since no specific values are given -- how can I determine an upper bound.
4. I am confused. I need numbers.
5. I guess no upper bound.

SET 5: $\{1, 1/2, 1/3, \ldots\}$

1. First noticed an infinite set and concluded no upper bound.
2. but, wait, it is decreasing $\therefore$ the largest number must be the upper bound = 1.

SET 6: The set of real numbers, R.

1. I imagine all types of numbers

   (rational, irrational, natural, etc.)

2. I see it going on forever.
3. No upper bound.

SET 11: The non-negative numbers.

1. Had trouble figuring out non-negative. First thought numbers to the left of zero.

   $\longleftarrow\!\!\!|_{\,0}$

2. Then thought of positive numbers, so no upper bound.

SET 13: The closed interval $[3, 5]$

1. Immediately thought upper bound.

SET 17: $\{\sqrt{2}\sqrt{3},\sqrt{4}\sqrt{5},\sqrt{6}\sqrt{7}, \ldots\}$

1. First noticed square root sign.
2. Then noticed progression with no upper bound.
3. Then asked if square root would increase to ∞
4. Decided it would, so no upper bound.

SET 18: $\{\text{Tan } x, \text{ where } x = \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots\}$

1. Originally thought no upper bound because of infinite progression.
2. But then realized that the value for tan = 0 at the given x values.
3. There is a upper bound.

SET 21: $\{1, 2, 4, 8, 16, \ldots\}$

1. Rapidly increasing numbers, so probably no upper bound.
2. No mental imagery.
3. Saw a geometric series, but of no importance. The set is infinite.

S17 E.T.

DEFINITION: Let E be a subset of the real numbers, R. An element, m, of R is said to be an upper bound of E if x is equal to or less than m for every x in E.

1. Discovered what the definition talks about, and what elements were in it. Then tried to relate the elements to each other.

2. m must be in R and greater or equal to x, which is in E.

$$E \leftarrow x \leq m \rightarrow R$$

PRACTICE PROBLEM: Let E be the set $\{1, 2, 3, 4\}$, i.e. the set consisting of the first four positive integers. Does E have an upper bound?

1. From the definition

$$E \leftarrow 1, 2, 3, 4 \leq m \rightarrow R$$

2. Therefore any number ≥ 4 is the upper bound.

SET 2: The set of natural numbers, N (i.e. the positive integers).

1. First drew mental image of the set

   $\{1, 2, 3, 4, \ldots\}$ and saw a number line going into infinity.

2. I decided that there is no m which is greater or equal to the largest x, $\therefore$ no upper bound.

SET 4: A finite set.

1. What is meant by a finite set?

   A specific number of points $\longrightarrow$ countable.

   For example: $\{1, 2, 3, 4\}$.

2. Since the set is countable it has an upper limit.

3. Remembered the term countable from discreet math or something.

SET 5: $\{1, 1/2, 1/3, \ldots\}$

1. I first tried to find a pattern, but could not find one.

2. Recognized "...".

3. Infinite set and therefore not bounded.

SET 6: The set of real numbers, R.

1. I am a little confused -- we are working with the real numbers (from definition).

2. If a set is a subset of itself then there is no upper bound.

   image:

SET 11: The non-negative numbers.

1. First read "negative numbers". A line which moves to the left instead of right.

2. Read again and saw line moving to the left

3. No.

SET 13: The closed interval $[3, 5]$.

1. Immediately related this to problem before.

Closed interval - finite - upper bound.

2. Used no imagery except noticed [ ].

<u>SET 17</u>: $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$

1. No.

2. Again "..." was seen. Infinite set — no upper bound.

<u>SET 18</u>: $\{\text{Tan } x, \text{ where } x = \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots\}$

1. I always had problems with tangent function.

2. The only images I see is the words,

   SIN COS

   but don't know how they relate to the π values.

<u>SET 21</u>: $\{1, 2, 4, 8, 16, \ldots\}$

1. Look at set.

2. Notice doubling $\rightarrow$ some type of geometric progression.

3. No upper bound/ no imagery.

S18 J.L.

DEFINITION: Let E be a subset of the real numbers, R. An element, m, of R is said to be an upper bound of E if x is equal to or less than m for every x in E.

1. I remember a definition similar to this for the upper bound.

2. $x \rightarrow E \rightarrow R$ and $m \rightarrow R$

   must hold that

   $m \geq x$

PRACTICE PROBLEM: Let E be the set $\{1, 2, 3, 4\}$, i.e. the set consisting of the first four positive integers. Does E have an upper bound?

1. I visualize the definition

   $1, 2, 3, 4 \rightarrow E \rightarrow R$ (reals)

   $m \geq 1, 2, 3, 4$

2. Reals are easy to define.

3. E was also easy to define.

4. Because I'd made an equation for m, I could plug the elements of E into x to convince myself that 4 was the upper bound.

SET 2: The set of natural numbers, N (i.e. the positive integers).

1. The numbers are positive and move to right.

   $1 \quad 2 \quad 3 \quad 4 \quad \longrightarrow$

2. Nothing can be to right of all of them, therefore upper bound exists.

SET 4: A finite set.

1. Same as practice problem

2. Yes.

3. Thought about answer immediate.

SET 5: $\{1, 1/2, 1/3, \ldots\}$

1. Another sequence.

2. $1, 1/2, 1/3, \ldots \rightarrow E \rightarrow R$

   $m \geq 1, 1/2, 1/3, \ldots$

3. Descending, therefore m exists.

4. Upper bound is anything equal to or greater than 1.

SET 6: The set of real numbers, R.

1. Clearly No!!!

2. It is just like N, moves to the right.

   no image

   no upper bound

SET 11: The non-negative numbers.

1. The word non-negative confusing.

2. Must be positive numbers.

   As before move off to the right.

3. No image.

   No upper bound.

SET 13: The closed interval $[3, 5]$

1. Knew the answer before w-iting.

2. Thought that whether the interval is open or closed is irrelevant.

SET 17: $\{\sqrt{2}\sqrt{3},\sqrt{4},\sqrt{5},\sqrt{6},\sqrt{7}, \ldots\}$

1. Another sequence.

2. Square root is unbounded $\longrightarrow \sqrt{n} = \infty$

3. Therefore no upper bound exist.

4. No image.

SET 18: $\{\text{Tan } x, \text{ where } x = \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots\}$.

1. This made me think about high school math.

2. Sequence.

   tan = y/x = 0

3. Upper bound exists.

SET 21: $\{1, 2, 4, 8, 16, \ldots\}$.

1. Some type of geometric sequence.

2. Obviously fast disappearing to right.

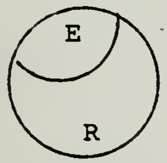
3. No upper bound.

   No image.

S19 L.N.

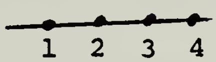
DEFINITION: Let E be a subset of the real numbers, R. An element, m, of R is said to be an upper bound of E if x is equal to or less than m for every x in E.

1.



2.

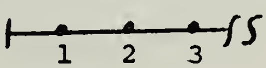
PRACTICE PROBLEM: Let E be the set $\{1, 2, 3, 4\}$, i.e. the set consisting of the first four positive integers. Does E have an upper bound?

1. 

4

Look to right for highest order number.

SET 2: The set of natural numbers, N (i.e. the positive integers).

1. 

etc.

2. Asked myself if natural was correct term.
   Drew line, checked to make sure I had not included zero.

3. Where is set 1?

SET 4: A finite set.

1. ....

...
...
...

$\{ 1, 2, 3, \ldots, n\}$

2. What does he want?

   I think of points in line or an array.

3. Since only specified number of points, upper bound exists and must be n.

4. Where is set 3?

<u>SET 5</u>: $\{1, 1/2, 1/3, \ldots\}$

1. ~~Rational-numbers-~~ -- not all there.

   infinite set

$$\{1/n\}_{n=1 \text{ to } \infty}$$

$$[1, 0] \Rightarrow 0$$

2. Decreasing, largest value is upper bound.

3. Are these questions or stimuli?

   Are they random? Is that why the set numbers vary?

<u>SET 6</u>: The set of real numbers, R.

1.  Re

   <u>solid</u> line

   usually think of Re at end of line.

2. Infinite, thus unbounded.

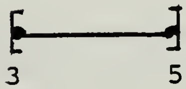
<u>SET 11</u>: The non-negative numbers.

1. 

   0, 1, 2, 3, ... etc.

2. Include zero this time.

<u>SET 13</u>: The closed interval $[3, 5]$.

1. 

2. Open interval uses ( ).

   Closed interval uses [ ]

3. 5

<u>SET 17</u>: $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$

1. Integers $>$ 1

   $\{\sqrt{n} \quad n=2 \to \infty$

2. Why so cryptic?

   I am looking for small details.

3. No bound $\longrightarrow$ infinity.

<u>SET 18</u>: $\{$ Tan x, where $x = \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots \}$

1. What goes on here?

2. What is he trying to do with this set?

3. What can he hope to gain from this experiment?

   Looks like a psych. experiment.

4. Why so much blank paper?

<u>SET 21</u>: $\{1, 2, 4, 8, 16, \ldots\}$

1. $\{2^n\}$ $n=0,1,2,\ldots$

2. Infinite set.

S20 T.M.

DEFINITION: Let E be a subset of the real numbers, R. An element, m, of R is said to be an upper bound of E if x is equal to or less than m for every x in E.

1. Very confusing. I start to panic.

2. Relate subset E to point m.

3. Look at "less than" or "equal to" part of definition.

PRACTICE PROBLEM: Let E be the set $\{1, 2, 3, 4\}$, i.e. the set consisting of the first four integers. Does E have an upper bound?

1. Yes.

SET 2: The set of natural numbers, N (i.e. the positive integers).

1. Saw this:

2. Not paying attention to the left, but infinite in both direction

3. Concluded no.

3. "Is N bounded?" No.

SET 4: A finite set

1. I don't see any numbers.

2. What happens if the finite set has infinite points

$[-\infty, +\infty]$

3. How can I assign an m to this.

<u>SET 5</u>: $\{1, 1/2, 1/3, \ldots\}$

1. I see a sequence going to infinity

2. Therefore must be bounded.

<u>SET 6</u>: The set of real numbers, R.

1. Automatic reaction to infinite set, thus could not define m.
2. No mental image.
3. Definition mentions R how does it relate to this set?

<u>SET 11</u>: The non-negative numbers.

1. Half-line in mind → positive.
2. Here, I realized no upper bound.

<u>SET 13</u>: The closed interval $[3, 5]$

1. First see the word interval, indicates no upper bound.
2. Infinite number of points.

   (infinite)

3. No.

SET 17: $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$

1. No upper bound.
2. I changed to more conventional numbers

   1.14, 1.492, 2, etc.

3. Then I was convinced the series ascended and was infinite.

SET 18: $\{\text{Tan } x, \text{ where } x = \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots\}$

1. No upper bound.
2. Like the previous problem -- I am convinced the series is ascending and is infinite.
3. No mental image.

SET 21: $\{1, 2, 4, 8, 16, \ldots\}$

1. The progression is ascending very rapidly to the right

   $\bullet\xrightarrow{\text{fast}}$

2. It is infinite $\therefore$ no upper bound.

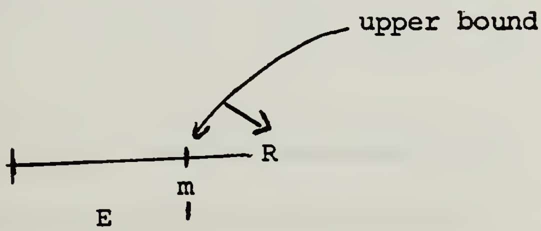
S21 A.C.

DEFINITION: Let E be the subset of the real numbers, R. An element, m, of R is said to be an upper bound of E if x is equal to or less than m for every x in E.

1. Image:

2. I try to relate the elements on the line.

   Draw:



PRACTICE PROBLEM: Let E be the set $\{1, 2, 3, 4\}$, i.e. the set consisting of the first four positive integers. Does E have an upper bound?

1. I see 4 points or elements.

2. Hazy, dark with white circles.

3. Finite.

4. Yes.

SET 2: The set of natural numbers, N (i.e. the positive integers).

1. Got side-tracked on definition.

2. Remember the term "countably infinite".

   The set numbers does approach infinity

3. I visualize myself counting the positive integers and saw there

would be no end to the counting.

4. No upper bound.

SET 4: A finite set.

1. This does not seem to fulfill all the requirements of definition.
2. But finite means an ending.

   If there is an end then there must be an m greater than the end.
3. Its finite $\therefore$ therefore upper bound.

SET 5: $\{1, 1/2, 1/3, \ldots\}$

1. The infinite series converges to 0.
2. The answer is the largest element of the set.

   m = 1 or greater.

SET 6: The set of real numbers, R.

1. I visualize a single straight line with no end in either direction.

2. I conclude there is no bound in either direction.

SET 11: The non-negative numbers.

1. What are non-negative numbers?
2. Answer: positive numbers.
3. Positive numbers go to infinity.
4. Therefore no bound (upper).
5. No image.

<u>SET 13</u>: The closed interval 3, 5

1. Closed interval is finite.

2. Is it?

   Draw:

3. No, interval is not finite.

   But closed means endpoint.

4. Yes, m=5

<u>SET 17</u>: $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$

1. Tried to determine if $\sqrt{}$ of numbers was increasing or decreasing.

2. Saw the dots.

3. I better determine if increasing or decreasing.

4. Increasing.

5. The set is infinitely large.

6. No upper bound.

<u>SET 18</u>: $\{$Tan x where x $= \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots\}$.

1. Tried to determine if tan x is increasing or decreasing.

2. I remember: $\tan x = \dfrac{\sin x}{\cos x}$

   But can't relate the π values to them.

3. Saw the dots.

4. The set is increasing.

5. The set is infinitely large.

6. No upper bound.

<u>SET 21</u>: $\{1, 2, 4, 8, 16, ...\}$.

1. As before, the set increases --- rapidly.
2. Saw the dots.
3. No upper bound.
4. No image, because obvious.

S22 J.T.

DEFINITION: Let E be a subset of the real numbers, R. An element, m, is said to be an upper bound of E if x is equal to or less than m for every x in E.

1. E $\{R\}$
2. E = $\{$x is an element$\}$
3. m $\geq$ x $\rightarrow$ R
4. The meaning of upper bound is ambiguous. I hope to figure it out after doing some problems.

PRACTICE PROBLEMS: Let E be the set $\{1, 2, 3, 4\}$, i.e. the set consisting of the first four positive integers. Does E have an upper bound?

1. E = 1, 2, 3, 4
2. 1, 2, 3, 4 are positive.
3. Read definition.
4. where does m fit?
5. m is a real, so is x.
6. Aha, m $\geq$ x, therefore 4 or greater.
7. Let's try next problem.

SET 2: The set of natural numbers, N (i.e. the positive integers).

1. E = $\{1, 2, 3, 4, \ldots\}$

   image: Think of positive integers and natural numbers.
2. Read definition.
3. The set goes to infinity. Does that mean if m = ∞ it is the upper bound?

4. Infinity is not a real number ∴ no upper bound.

5. I am still confused with definition.

SET 4: A finite set.

1. This is the same as the practice problem.

2. Yes, upper bound.

3. I didn't use definition or pictures for this set.

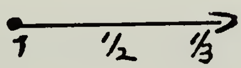
SET5: $\{1, 1/2, 1/3, \ldots\}$.

1. This set is also like the precious problem.

2. Turn to definition.

3. Confused.

4. Draw.

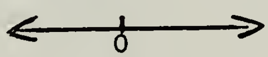


5. It is infinite and therefore there must be no m, no upper bound.

SET 6: The set of real numbers, R.

1. Infinite series in both direction, not just to the right. Zero as center.

2. image:



3. Getting the hang of definition.

   If no m or infinite no upper bound.

SET 11: The non-negative numbers.

1. Infinite series to the left (no image).

2. Same as previous problem.

   No m and infinite means no upper bound.

3. The problems are going quicker now.

<u>SET 13</u>: The closed interval $[3, 5]$

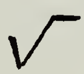
1. This is finite.

2. Upper bound.

<u>SET 17</u>: $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$

1. Infinite series.

   images:



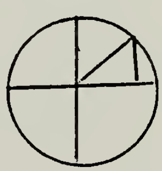
2. numbers seem to be increasing.

3. As before, no m, no upper bound.

<u>SET 18</u>: $\{\text{Tan } x$, where $x = \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots\}$

1. Trig function are cyclic

   Image:



2.

3. Since infinite sequence, graph of tan function goes to infinity somewhere.

4. No upper bound.

SET 21: $\{1, 2, 4, 8, 16, \ldots\}$

1. Saw familiar pattern, 2^n
2. Saw dots.
3. Quickly decided it was infinite.
4. No upper bound.
5. No imagery.

S23 J.B.

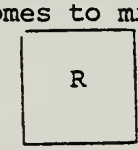
DEFINITION: Let E be a subset of the real numbers, R. An element, m, of R is said to be an upper bound of E if x is equal to or less than m for every x in E.

1. First I have to think what the term subset means.

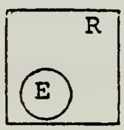
2. I think about a set, which is a collection of things:

(i.e. red, blue, green)

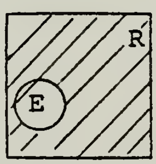
3. A Venn diagram comes to mind, using real numbers as the universe.



4. A subset, must therefore be a small set, within R.



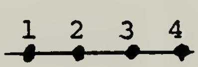
5. Looking at the Venn diagram, I think about the upper bound as being everything in the universe R with the exception of the area within the small circle.



PRACTICE PROBLEM: Let E be the set $\{1, 2, 3, 4\}$, i.e. the set consisting of the first four positive integers. Does E have an upper bound?

1. This subset contains a finite set of numbers.

2. They are increasing in size.

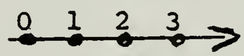


3. Since it is finite the largest number is 4. The upper bound = 4.

<u>SET 2</u>: The set of natural numbers, N (i.e. the positive integers).

1. First thing that comes to mind is -- what is a natural number.

2. The series is increasing



3. Its an infinite series going to infinity (∞)

4. No upper bound.

<u>SET 4</u>: A finite set

1. what is a finite set?

2. Its a specified number of things.

or

$[a, b], [c, d]$

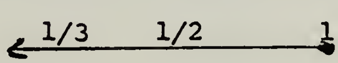
3. Therefore it has an upper bound.

<u>SET 5</u>: $\{1, 1/2, 1/3, \ldots\}$

1. Look at the dots indicating a infinite set.

2. Infinite sets do not have upper bounds.

3. Looking at the numbers they are decreasing going towards 0.

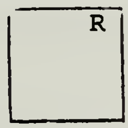


4. Upper bound of 1.

<u>SET 6</u>: The set of real numbers, R.

1. What is the set of real numbers.

2. Remember the Venn diagram I used in the definition.



3. No upper bound, since it goes to infinity.

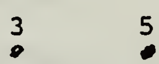
<u>SET 11</u>: The non-negative numbers.

1. This brings back set 2 (natural # - positive integers)

2. It goes to infinity, therefore <u>no</u> upper bound.

<u>SET 13</u>: The closed interval $[3, 5]$

1. Immediate reaction of upper bound of 5, since it is a finite set.



<u>SET 17</u>: $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$

1. First looked at dots, indicating an infinite set, therefore no upper bound.

   i.e. $\sqrt{4} = 2$

   $\sqrt{9} = 3$

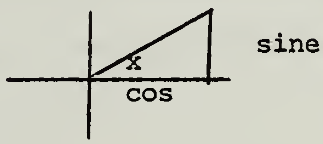
3. Indicates the larger the number gets -- the larger the square of it.

4. No upper bound goes to ∞

<u>SET 18</u>: Tan x, where x = , 2 , 3 , 4 , 5 , 6 , ...

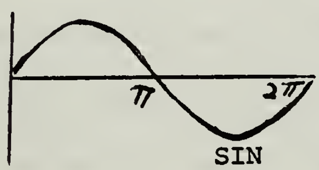
1. Had to remember what tangent is.

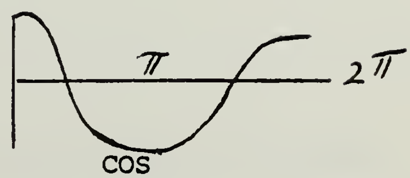
2. Drew picture to figure it out.



$$\tan = \frac{\sin}{\cos}$$

3. What where the values of sin & cos at π, 2π, etc.





4. $\tan \quad = \quad \frac{\sin}{\cos} \quad = \quad \frac{0}{-1} \quad = \quad 0$

$\tan 2 \quad = \quad \frac{\sin 2}{\cos 2} = \quad \frac{0}{1} \quad = \quad 0$

5. Therefore the set is 0, 0, 0, ...

6. Eventhough infinite set the numbers are constant 0. Therefore upper bound of 0.

<u>SET 21</u>: $\{1, 2\ 4, 8, 16, \ldots\}$

1. Its an infinite series -- does it have an upper bound.

2. Equation is $2^{(n-1)}$

3. When n becomes so large the -1 will have no effect.

4. Increasing set and since n can go to infinity -- no upper bound.

2. This is true for all finite sets.

SET 5: $\{1, 1/2, 1/3, \ldots\}$

1. Saw the form 1/n.

2. Looked at consecutive pairs; saw that denominator was increasing.

3. Infinite set and therefore upper bound.

   (no image)

SET 6: The set of real numbers, R.

1. From memory R has no upper bound.

   I don't think I used the image.

2. I hesitate to use too many images because they are unreliable.

SET 11: The non-negative numbers.

1. I think what it means to be non-negative, getting an image of ≥ 0. Again I see an infinite set, and know that it is unbounded.

2. I note that it is bounded below.

SET 13: The closed interval [3, 5]

1. An interval means it is infinite, but also it is bracketed on both sides

2. In this problem the infinite set is bounded by 3 and 5, therefore it has an upper bound.

S24 S.M.

DEFINITION: Let E be a subset of the real numbers, R. An element, m, of R is said to be an upper bound of E if x is equal to or less than m for every x in E.

1. $m \in R$

2. $x \leq m$

3. The above equation is what I thought about when reading the definition.

PRACTICE PROBLEM: Let E be the set $\{1, 2, 3, 4\}$, i.e. the set consisting of the first four positive integers. Does E have an upper bound?

1. 4 elements in the set (finite set).

2. Therefore there is an upper bound.

SET 2: The set of natural numbers, N (i.e. the positive integers).

1. I focus on a particular set of numbers.

   1 2 3 4 ... etc

2. I can't tell the last element of E, since the number go to infinity.

3. I wonder if these do have upper bound?

4. I can't see an end to this set - so no upper bound.

SET 4: A finite set.

1. Concrete at end and beginning.

   Draw:

   $\{a, b, c, d, e, f, g\}$

   If I use above set where a b c d e f g, then

   g is the upper bound.

3. Confused, what would be the answer if the question was:

   The closed interval $[-\infty \ , +\infty]$

4. My answer for this problem is upper bound.

<u>SET 17</u>: $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots\}$

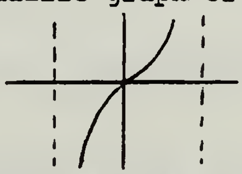
No, realized set was infinite.

1. Ignored $\sqrt{\ }$

2. Saw dots.

<u>SET 18</u>: $\{$ Tan x, where x $= \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots$

1. I tried to visualize graph of tan x:



2. Disregarded graph, because it didn't tell me what the values were at π, 2π, etc.

3. Went to mathematical form:

$$\tan = \frac{\sin}{\cos}$$

4. Not sure what the values are for π and 2π

5. See dots.

6. The set is infinite, $\therefore$ no upper bound.

7. I did not really solve the tan problem.

<u>SET 21</u>: $\{1, 2, 4, 8, 16, \ldots\}$

1. No, realized an infinite geometric increasing set.
   no image (automatic)

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